Hilbert-Huang Transform:

A New Method for Nonstationary and Nonlinear Time Series Analysis

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Data Analysis = Information Extraction

- Data is the only connection between us and the realty.
- All our information is contained in the data.
- Data analysis is the means to extract information form the data.
- Data analysis is mechanical; result interpretation is the key to yield information.
- Unless we have clear understanding of the underlying processes, data analysis should not be based on *a priori* basis methods.
- Adaptive basis is the best approach to extract the maximum amount information.
- Hilbert-Huang Transform (HHT) is based on an adaptive approach.

The Main Data Analysis Tasks

- Distribution: global properties limited to homogeneous population only; HHT can help extract component with homogeneous scale.
- Filtering: mostly Fourier based in frequency space; HHT is a nonlinear time scale based filter.
- Regression: fit data to an *a priori* functional; HHT fits adaptively with spline.
- Correlation: need to detrend; HHT offers adaptive detrend.
- Spectral Analysis: time-frequency representation; HHT for data from nonlinear and nonstationary processes.
- Prediction: stationary processes; HHT could help here too by provide band-limited components fro easier prediction.

History of HHT

1998: The Empirical Mode Decomposition Method and the Hilbert Spectrum for Non-stationary Time Series Analysis, *Proc. Roy. Soc. London*, A454, 903-995.

The invention of the basic method of EMD, and Hilbert transform for determining the Instantaneous Frequency and energy.

1999: A New View of Nonlinear Water Waves – The Hilbert Spectrum, *Ann. Rev. Fluid Mech.* 31, 417-457.

Introduction of the intermittence in EMD decomposition.

2003: A confidence Limit for the Empirical mode decomposition and the Hilbert spectral analysis, *Proc. of Roy. Soc. London*, A459, 2317-2345.

Establishment of a confidence limit without the ergodic assumption.

2004: A Study of the Characteristics of White Noise Using the Empirical Mode Decomposition Method, *Proc. Roy. Soc. London*, A465, 1597-1611.

Defined statistical significance and predictability for IMF from EMD.

2005: On Instantaneous Frequency, *Proc. Roy. Soc. London*, (Under review)

Removal of the limitations posted by Bedrosian and Nuttall theorems for Instantaneous Frequency computations.

2005: Ensemble Empirical Mode Decomposition: A Noise Assisted Data Analysis Method. *Proc. Roy. Soc. London*, (Under review)

Made EMD totally adaptive without subjective intermittence criteria.

2005: On the Trend and Detrend and Variability for nonstationary and nonlinear data. *Proc. Roy. Soc. London*, (Under review)

Determining the trend adaptively without a priori functional form.

My Prejudice

SEMINAR NOTICE

Mode analysis of non-stationary, random systems;
the overthrow of Fourier analysis.

Dr Norden Huang,
NASA Goddard Space Flight Center

Modern techniques for the analysis of non-stationary random systems and the identification of embedded characteristic structures include wavelet analysis and Hilbert transforms, each of which has its own limitations. A new and simple method applicable to non-linear systems will be described; it involves an effectively finite and often small number of discrete modes and gives sharp identification of embedded structures. Examples will be given of synthetic records analysed by various methods, and of real time series of non-linear systems, such as surface waves, tidal records and low-frequency oceanic oscillations; this new technique gives much simpler and more revealing interpretations than conventional methods do.

12 noon, Mon. 5 February, 304 Olin, JHU

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Motivations

- Physical processes are mostly nonstationary
- Physical Processes are mostly nonlinear
- Data from observations are invariably too short
- Physical processes are mostly non-repeatable.

◡ Ensemble mean impossible, and temporal mean might not be meaningful for lack of ergodicity. Traditional methods inadequate.

Available Data Analysis Methods for Nonstationary (but Linear) time series

- Various probability distributions
- Spectral analysis and Spectrogram
- Wavelet Analysis
- Wigner-Ville Distributions
- Empirical Orthogonal Functions aka Singular Spectral Analysis
- Moving means
- Successive differentiations

Available Data Analysis Methods for Nonlinear (but Stationary and Deterministic) time series

- Phase space method
  - Delay reconstruction and embedding
  - Poincaré surface of section
  - Self-similarity, attractor geometry & fractals
- Nonlinear Prediction
- Lyapunov Exponents for stability

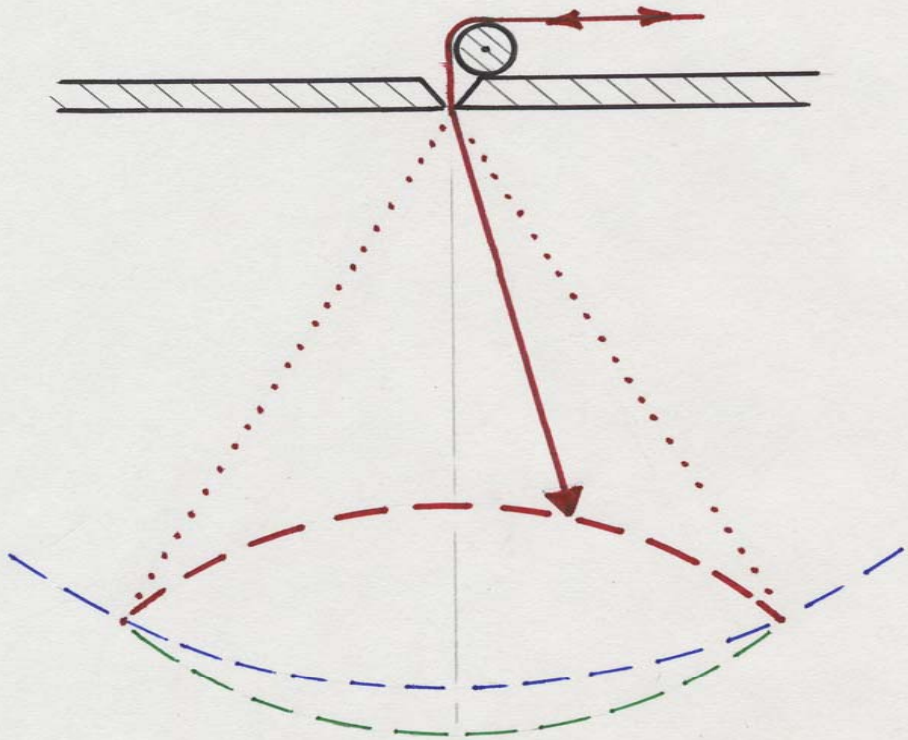
Characteristics of Data from Nonlinear Processes

$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = \gamma \cos \omega t$$

$$\Rightarrow \quad \frac{d^2 x}{dt^2} + x\left(1 + \varepsilon x^2\right) = \gamma \cos \omega t$$

$\Rightarrow$ *Spring with position dependent constant,*
intra – wave frequency modulation;
therefore, we need instantaneous frequency.

Duffing Pendulum



$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = a \cos \gamma t$$

$$\frac{d^2 x}{dt^2} + x(1 + \varepsilon x^2) = a \cos \gamma t$$

Hilbert Transform : Definition

$$\text{For any } x(t) \in L^p \ ,$$

$$y(t) = \frac{1}{\pi} \wp \int_{\tau} \frac{x(\tau)}{t-\tau} \, d\tau \ ,$$

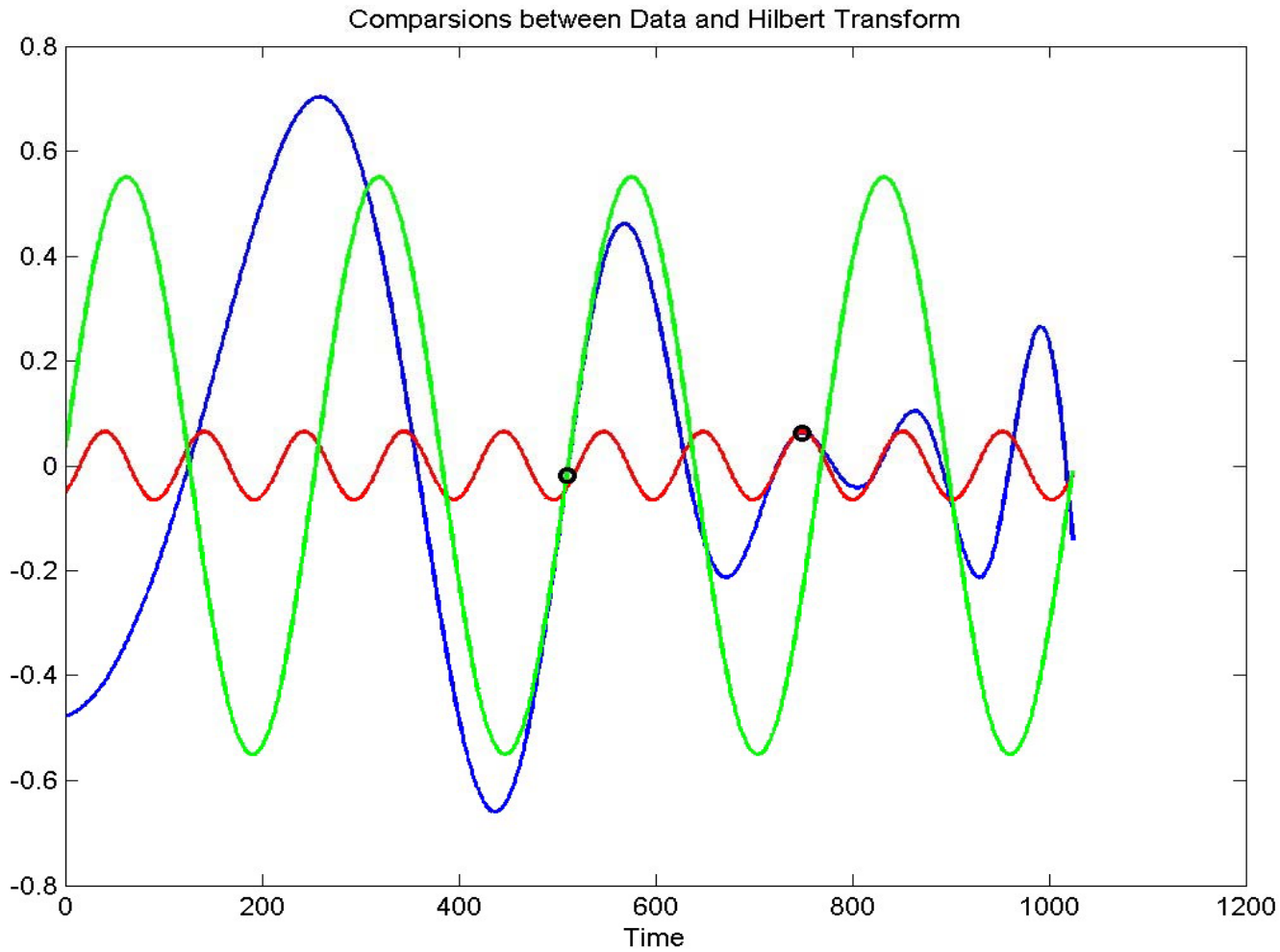
$$\text{then, } x(t) \text{ and } y(t) \text{ are complex conjugate :}$$

$$z(t) = x(t) + i\, y(t) = a(t)\, e^{i\theta(t)} \ ,$$

$$\text{where}$$

$$a(t) = \left(x^2 + y^2\right)^{1/2} \text{ and } \theta(t) = \tan^{-1} \frac{y(t)}{x(t)} \ .$$

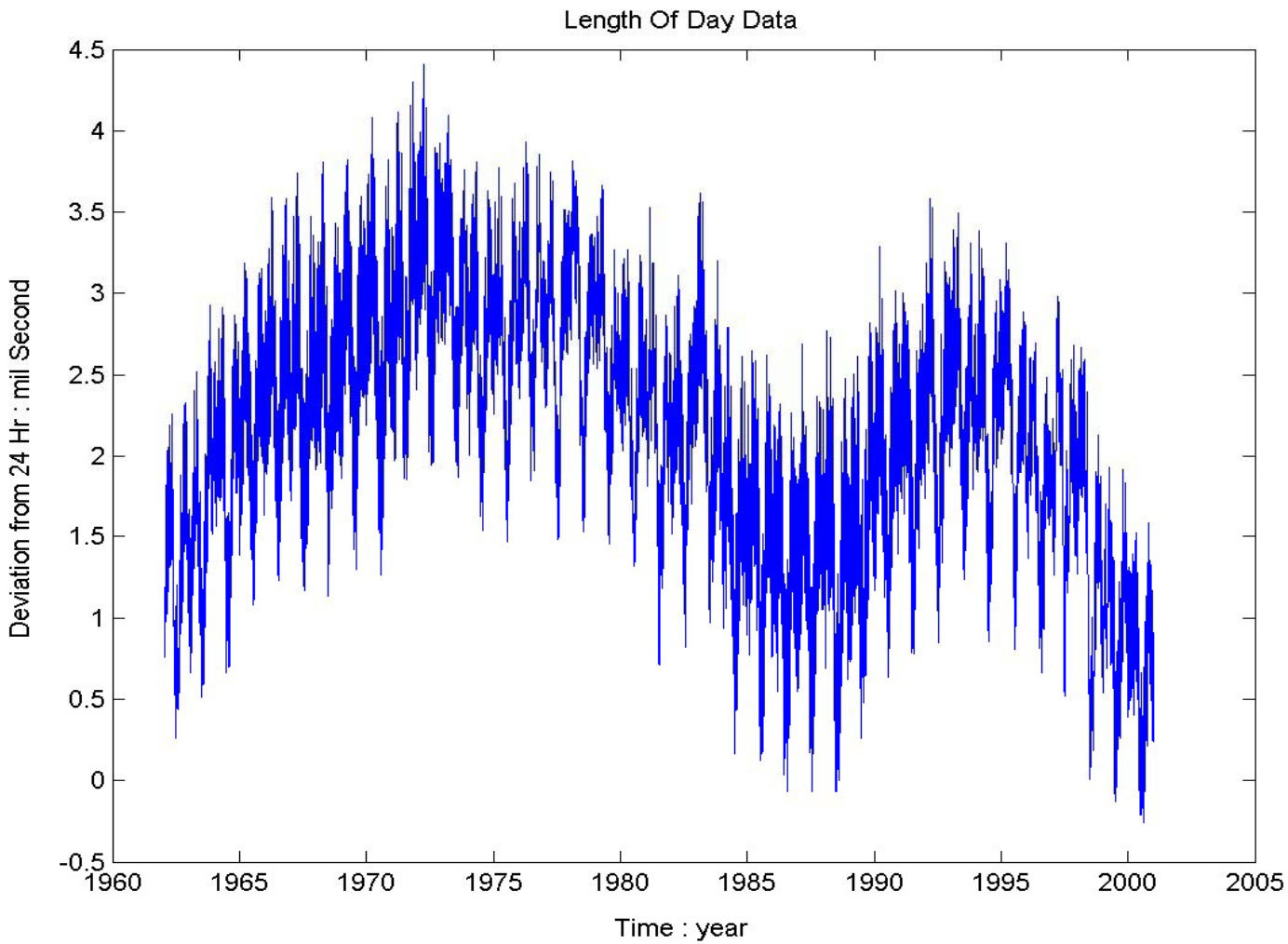
Hilbert Transform Fit



The Traditional View of the Hilbert Transform for Data Analysis

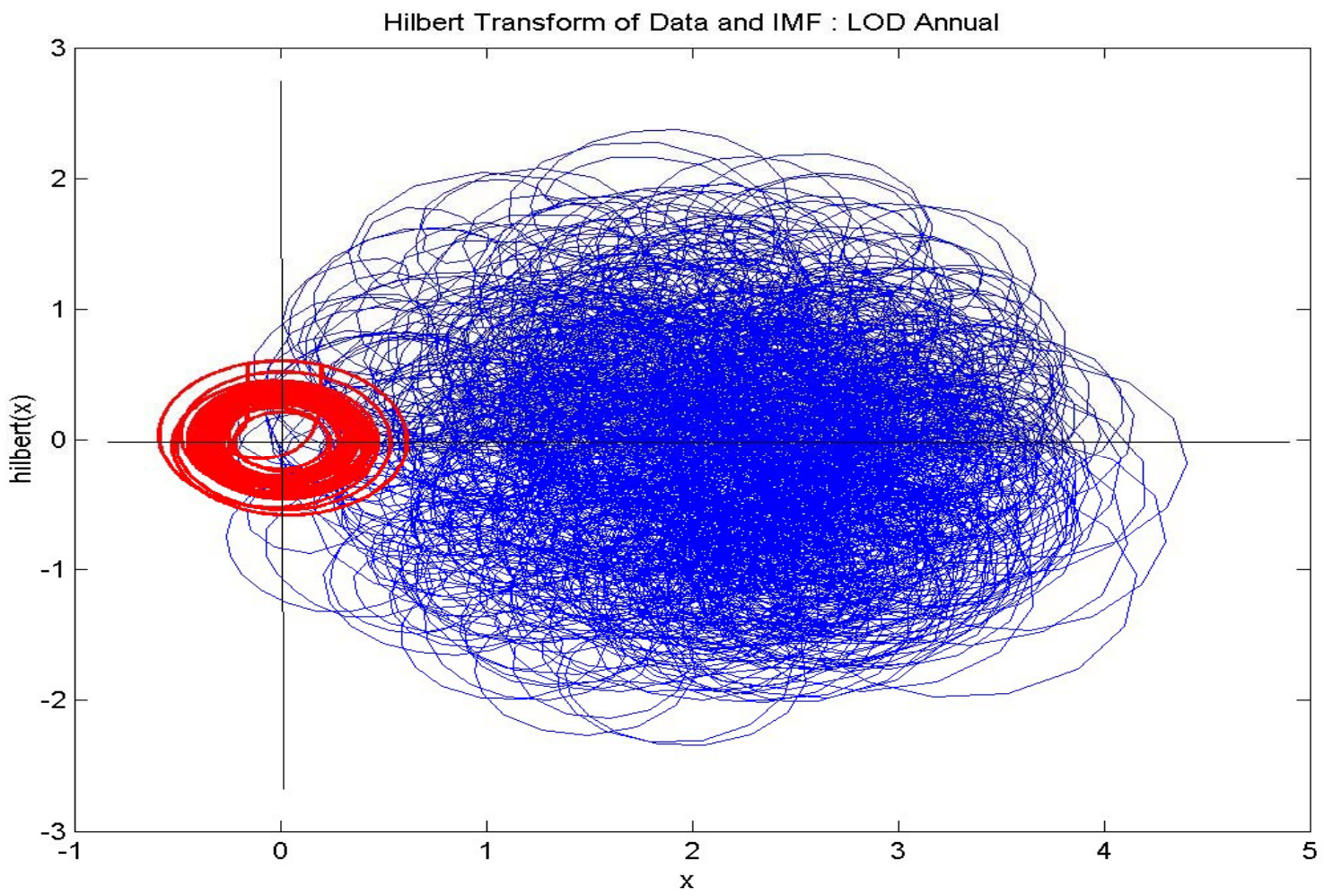
Traditional View

a la Hahn (1995) : Data LOD



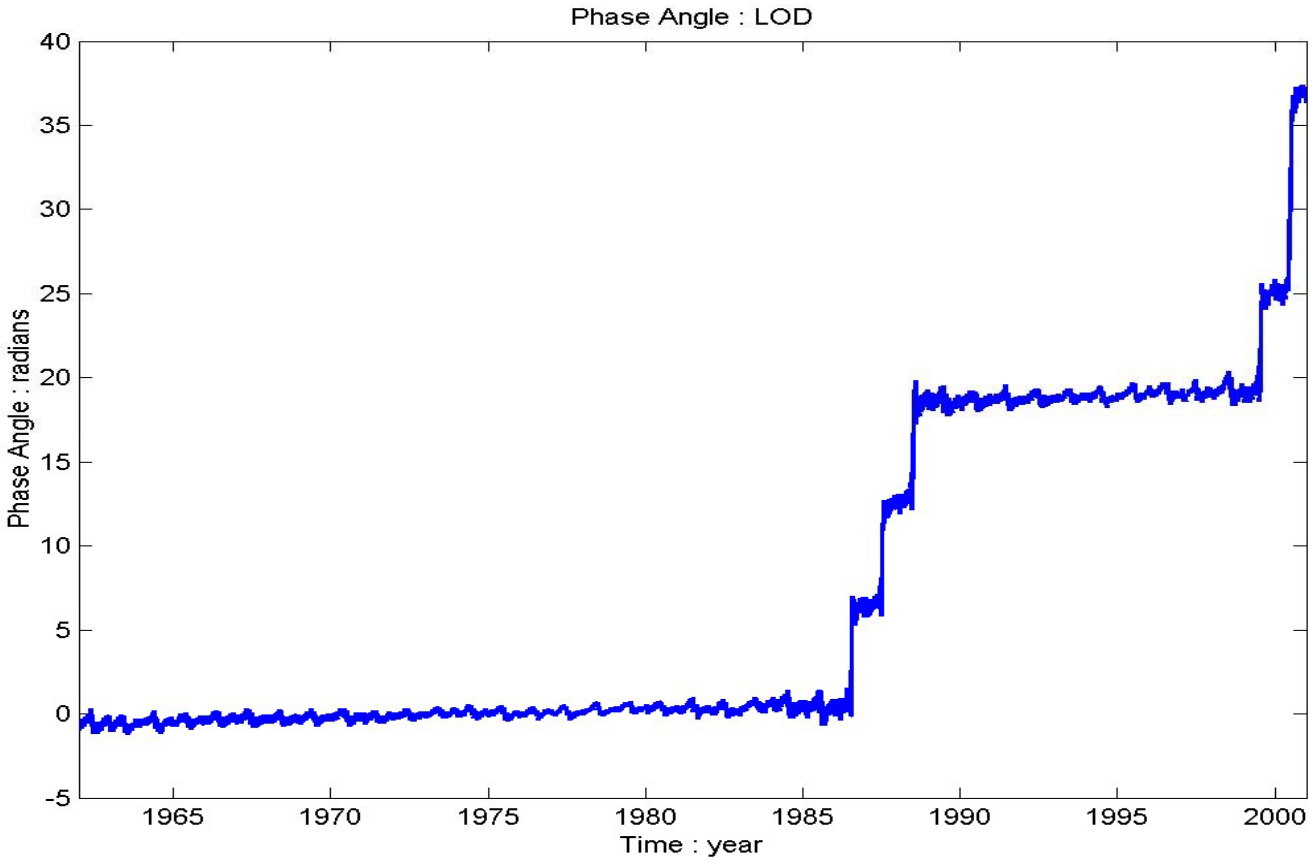
Traditional View

a la Hahn (1995) : Hilbert



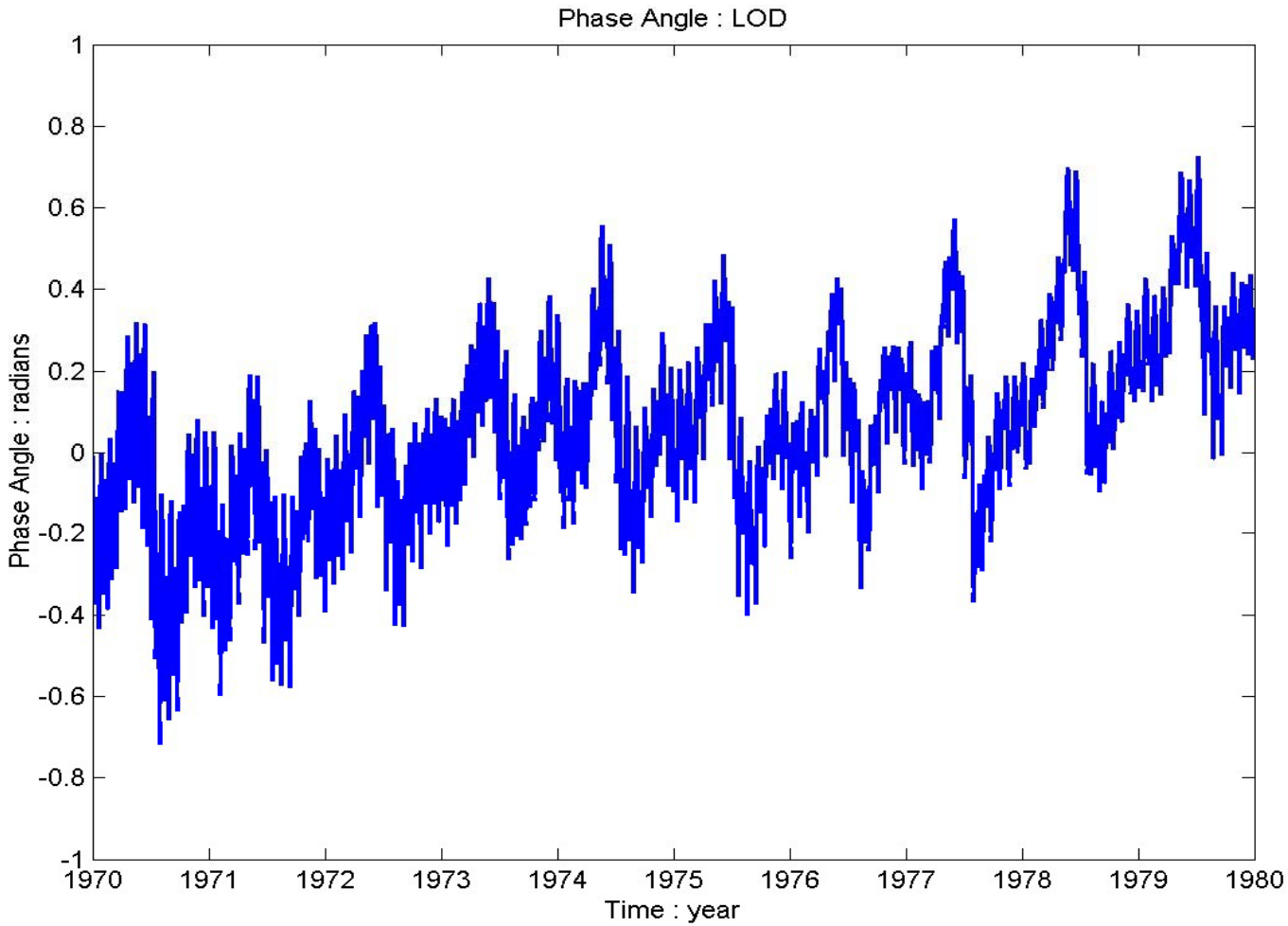
Traditional View

a la Hahn (1995) : Phase Angle



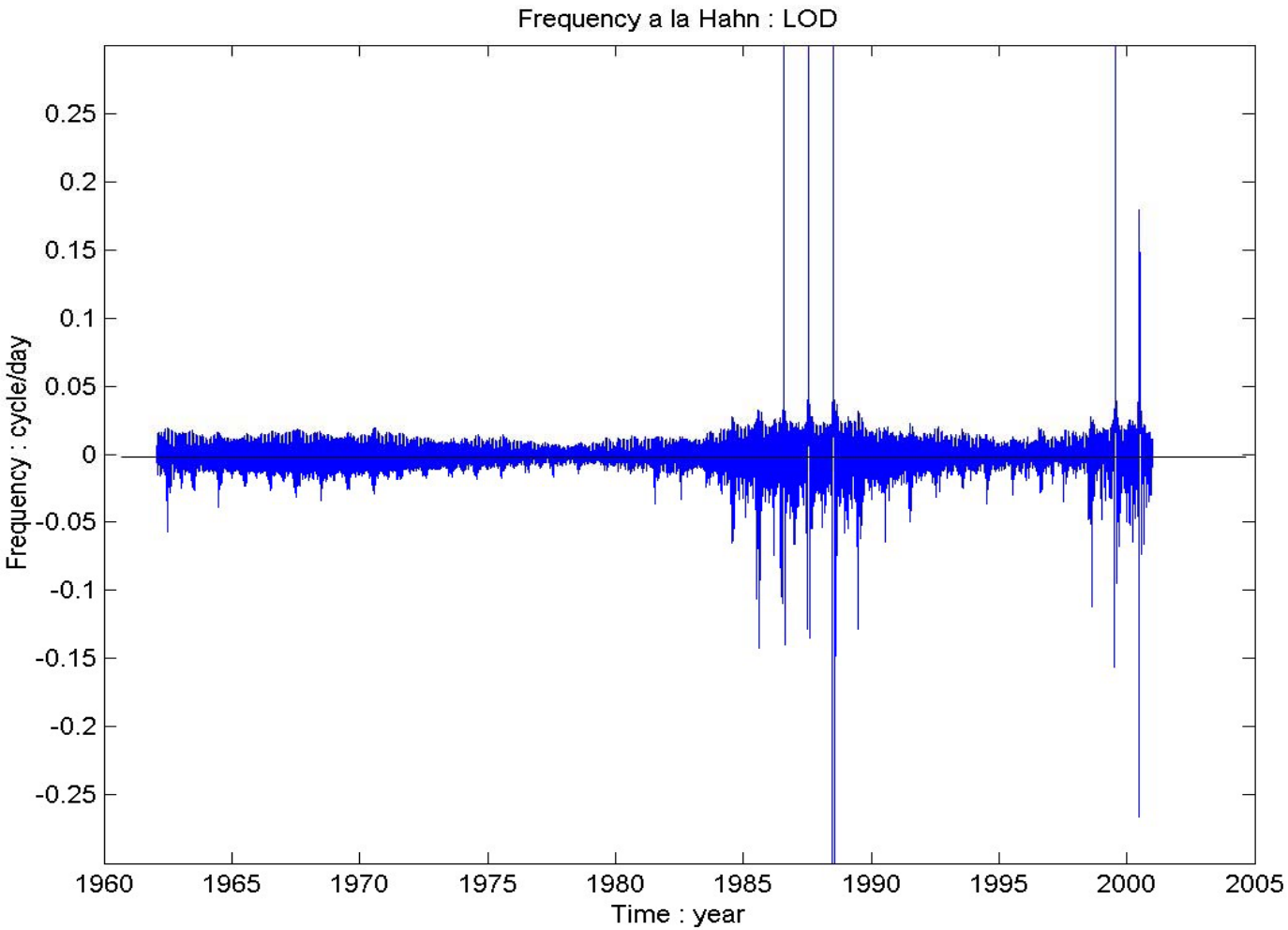
Traditional View

a la Hahn (1995) : Phase Angle Details



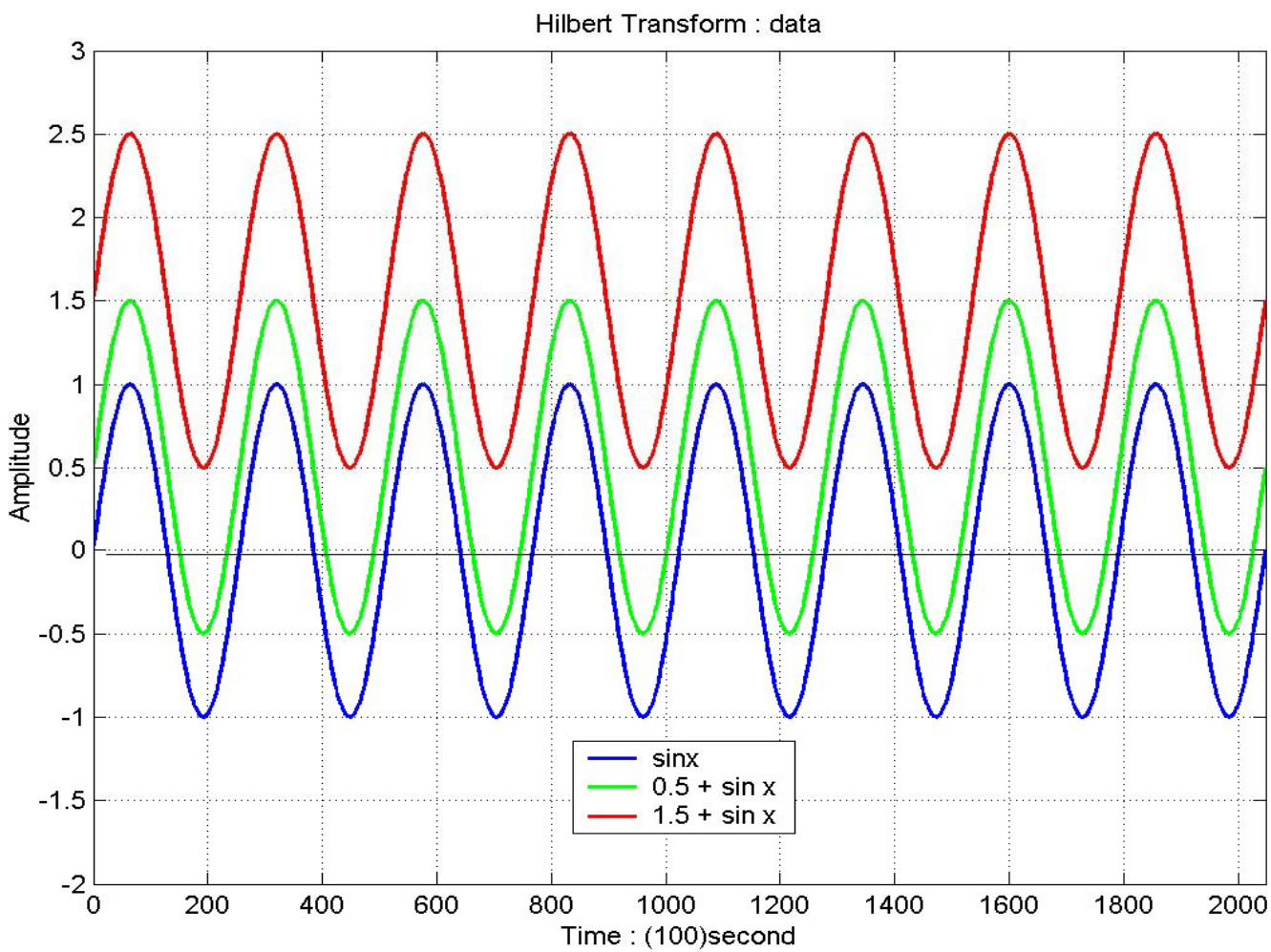
Traditional View

a la Hahn (1995) : Frequency



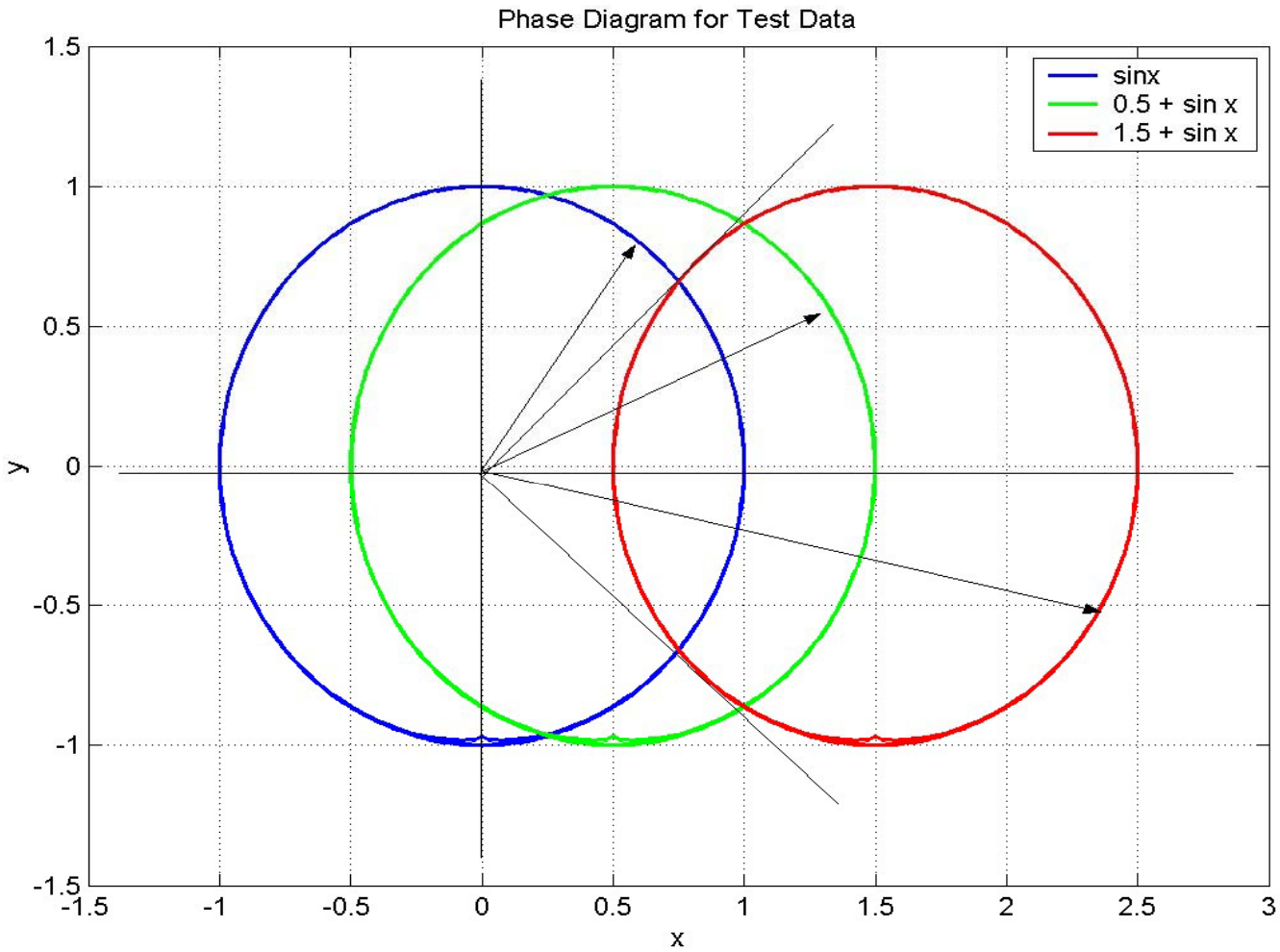
Why the traditional view does not work?

Hilbert Transform *a cos ❑ + b* : *Data*



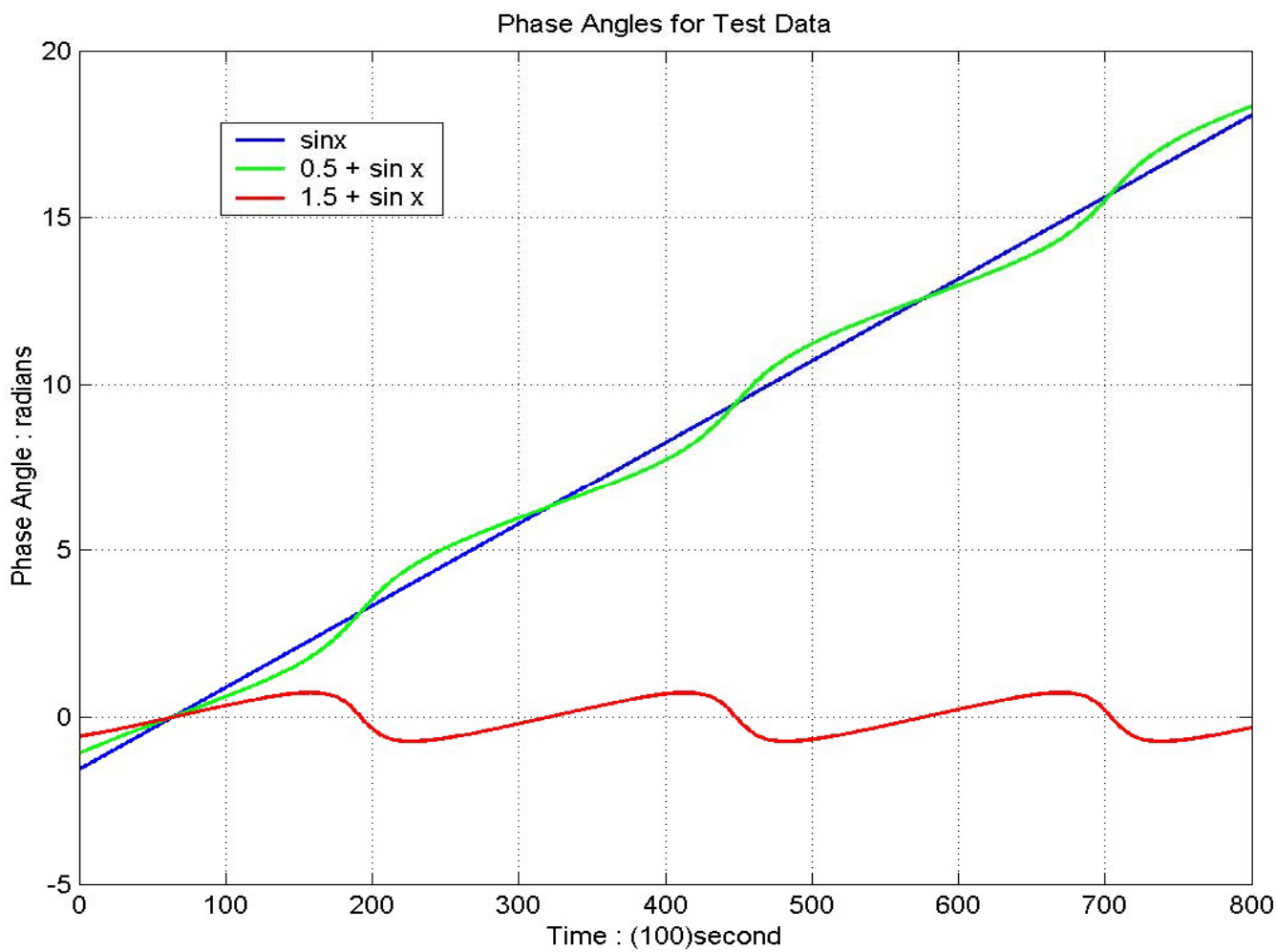
Hilbert Transform *a cos ❑ + b :*

Phase Diagram



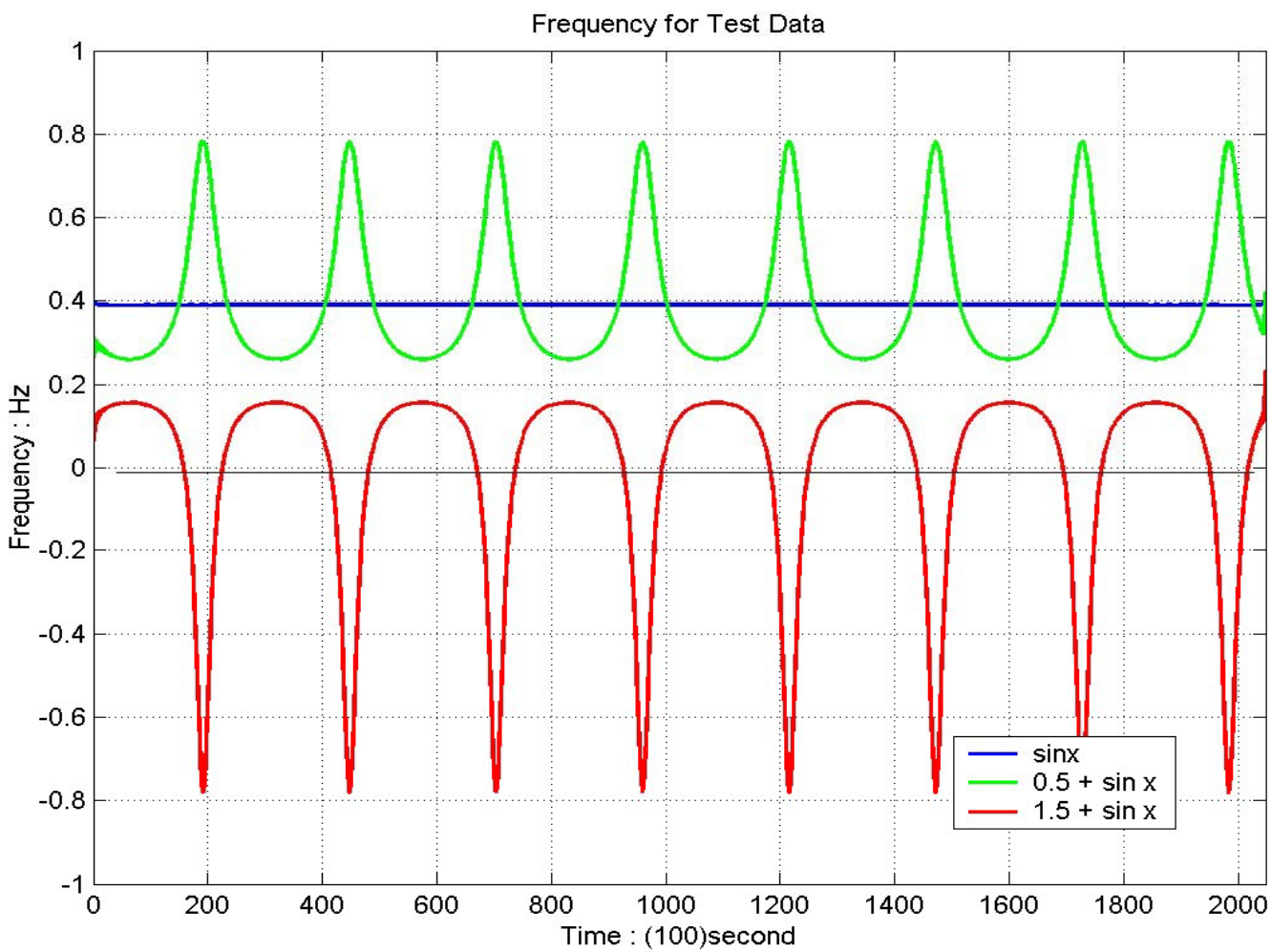
Hilbert Transform *a cos ❑ + b :*

Phase Angle Details



Hilbert Transform *a cos* ❑ + *b* :

Frequency

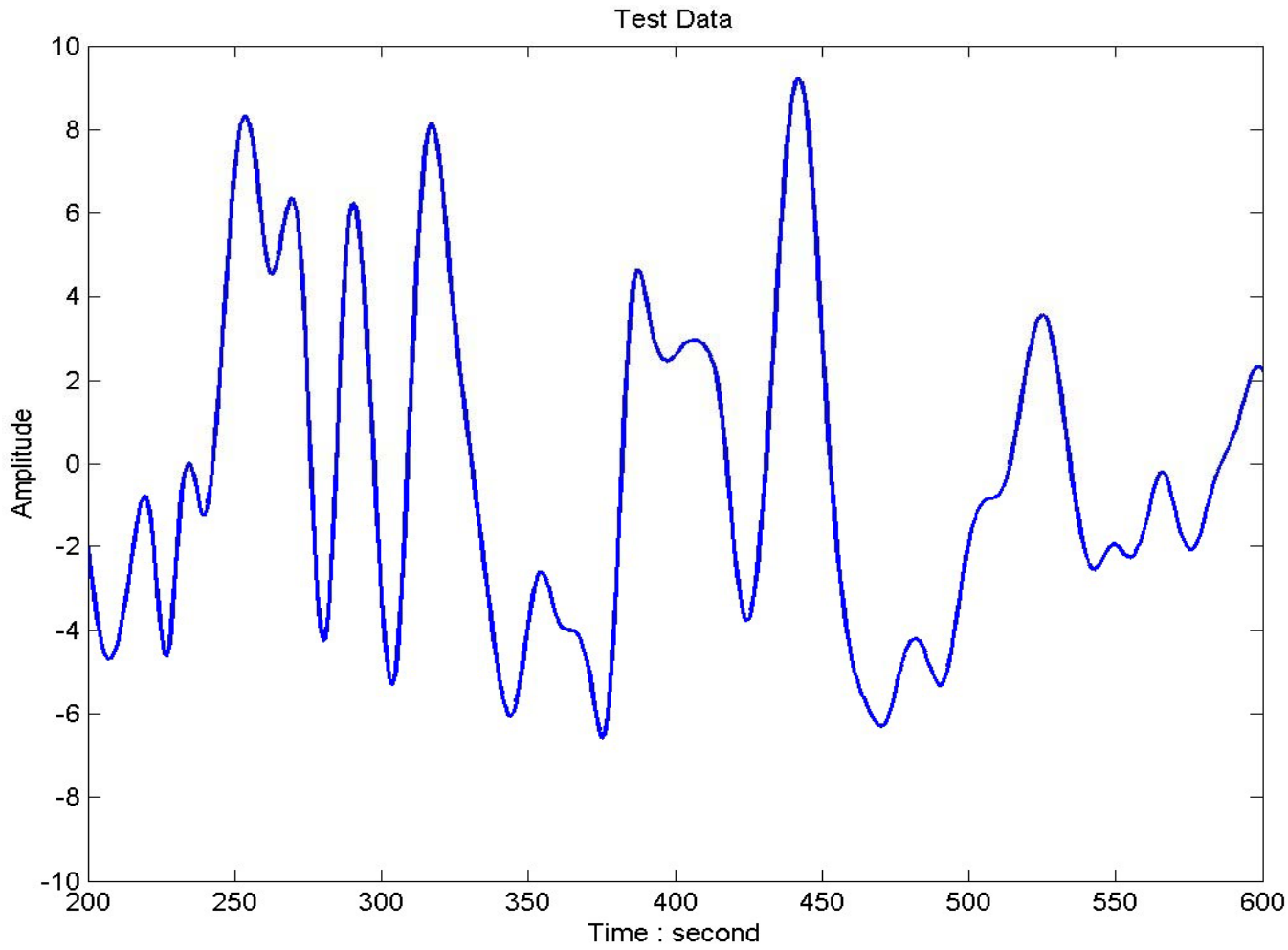


The Empirical Mode Decomposition Method and Hilbert Spectral Analysis

Sifting

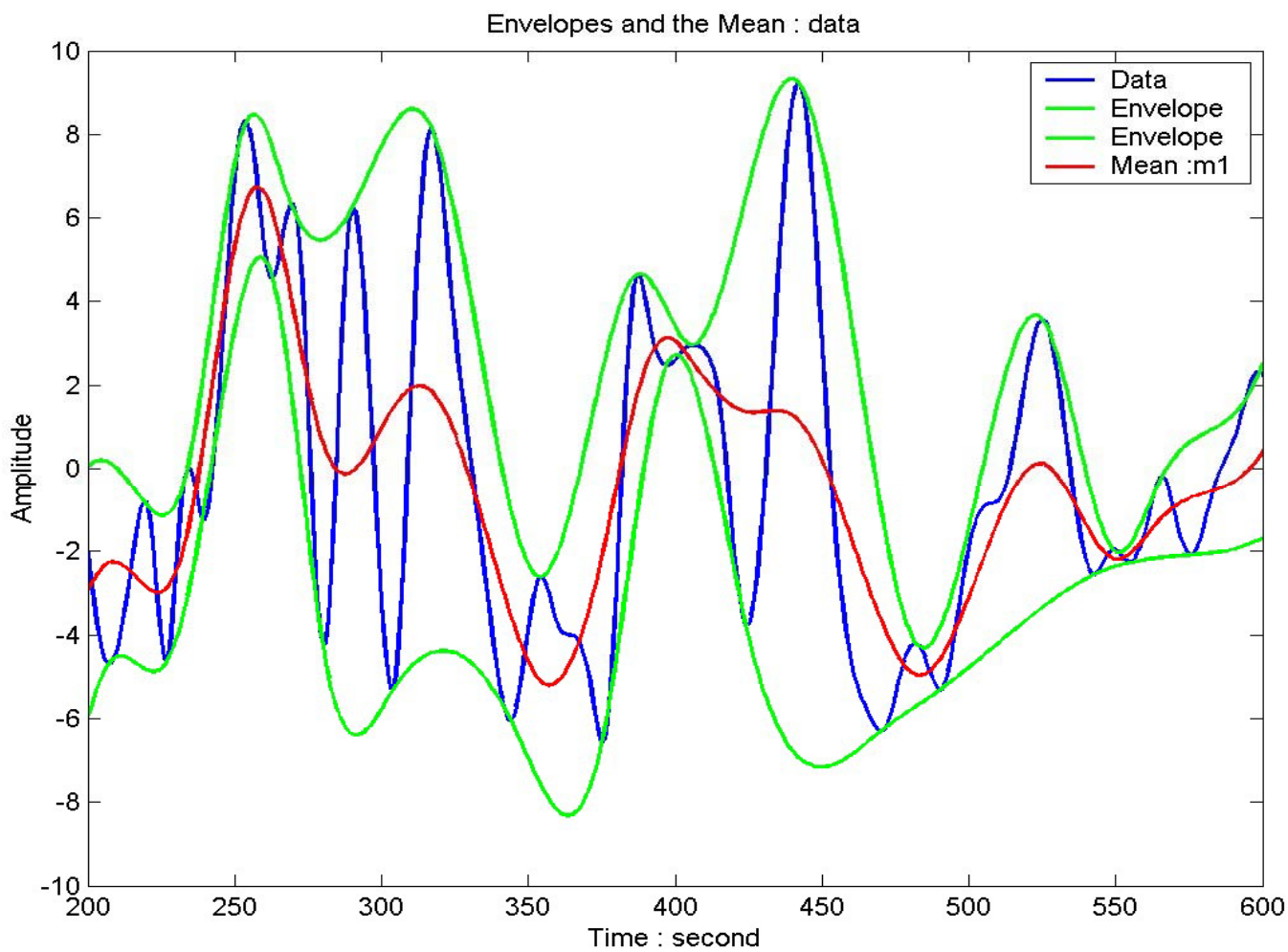
Empirical Mode Decomposition:

Methodology : Test Data

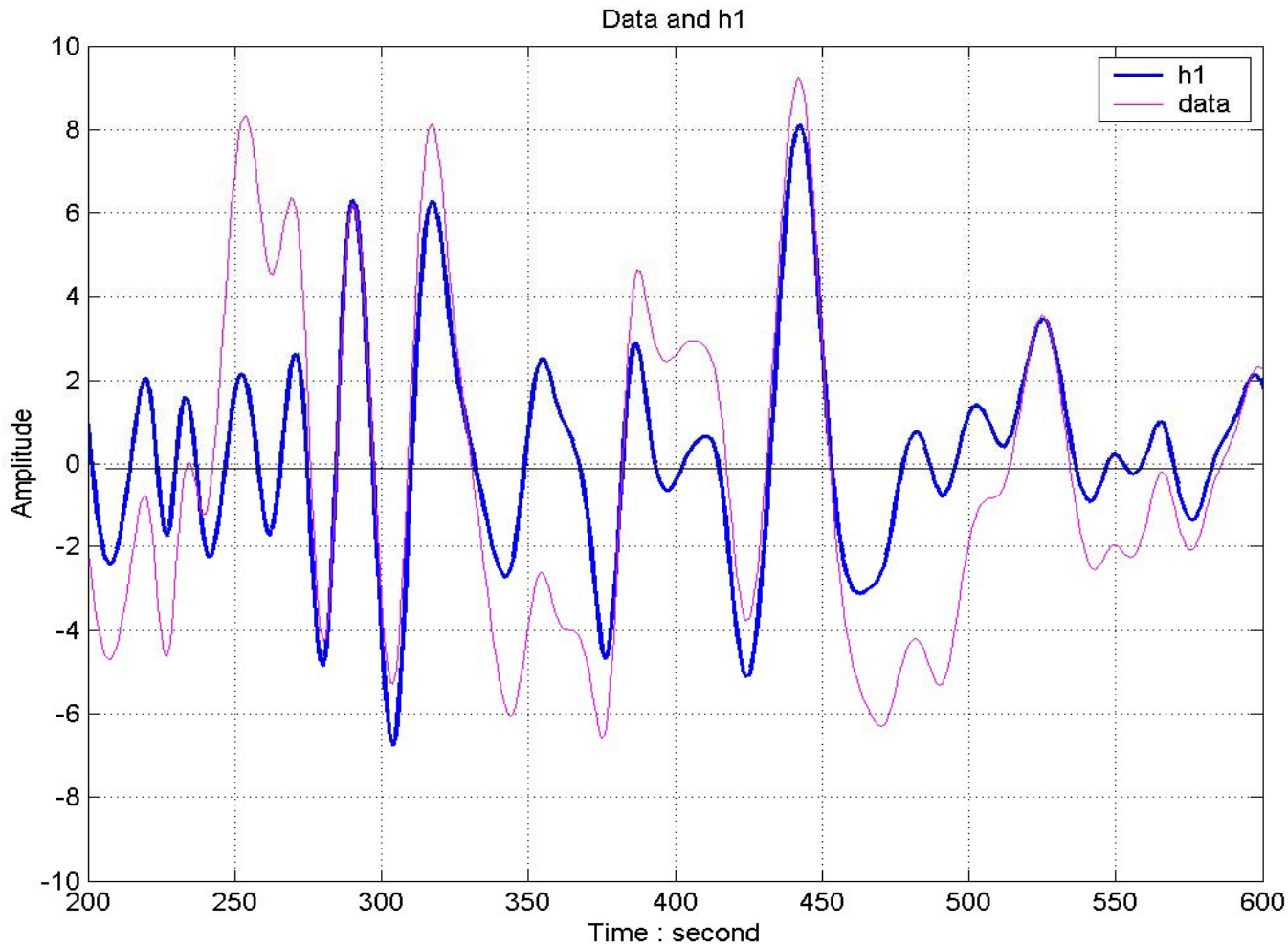


Empirical Mode Decomposition:

Methodology : data and m1

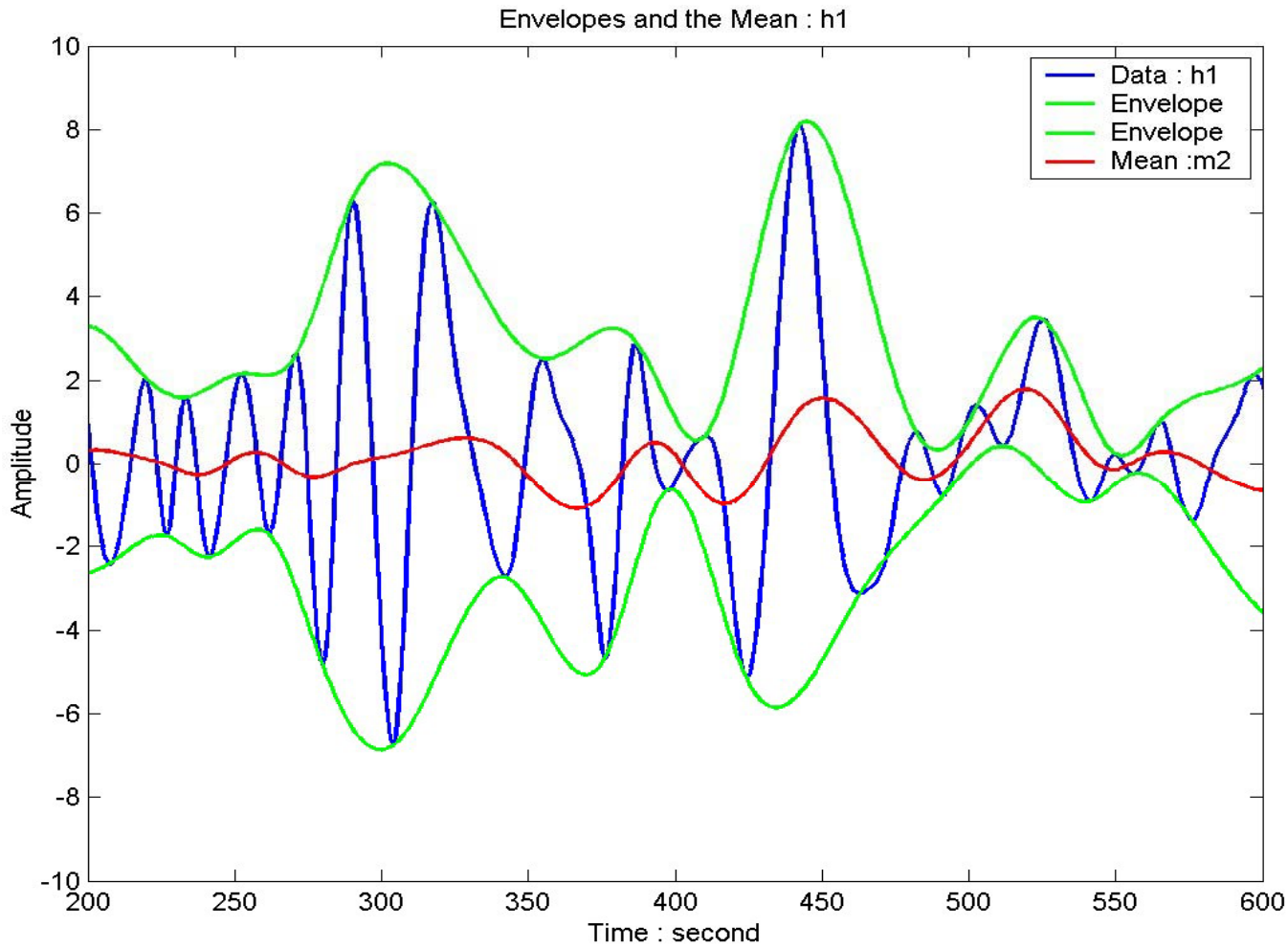


Empirical Mode Decomposition:
Methodology : data & h1



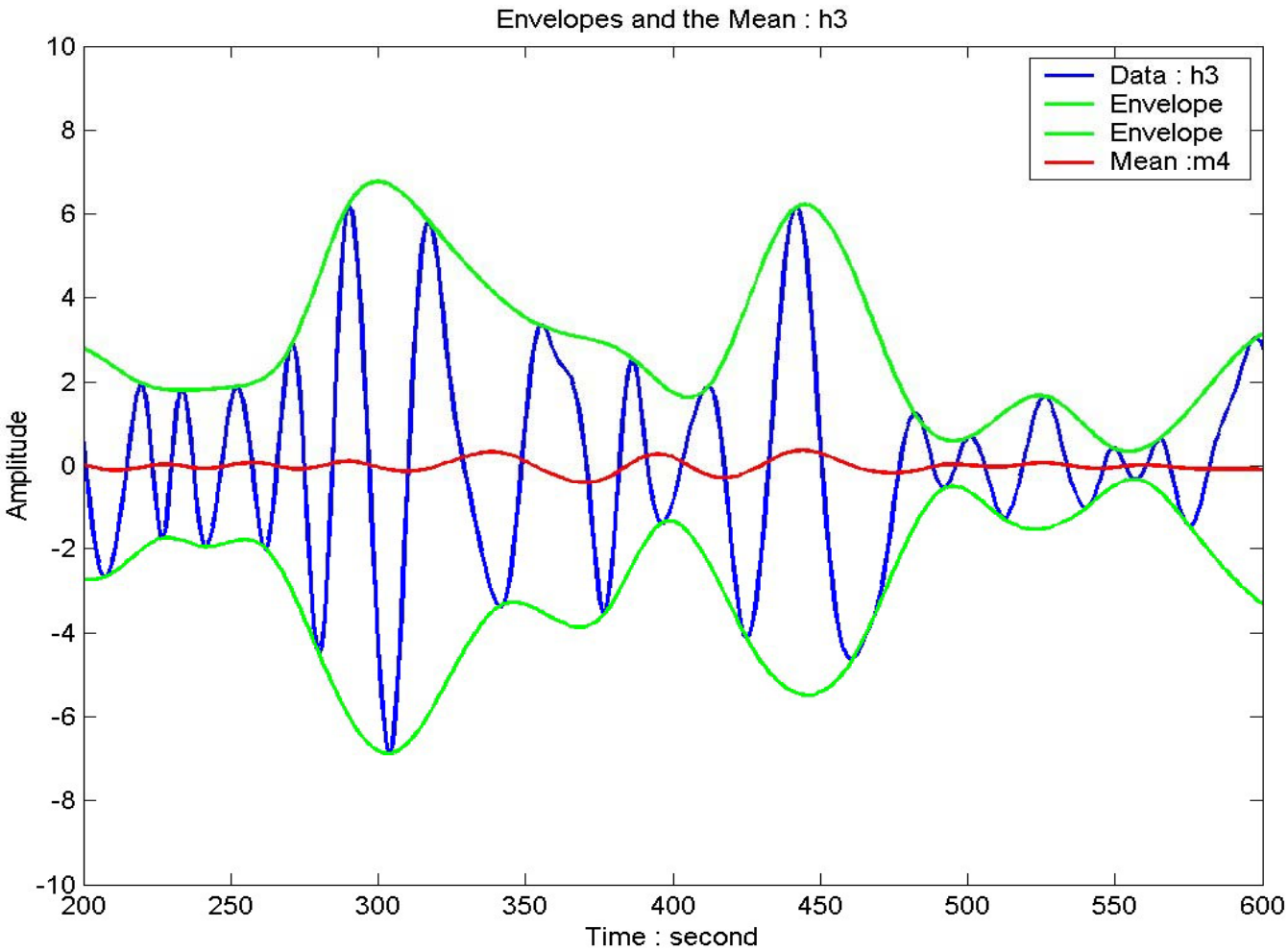
Empirical Mode Decomposition:

Methodology : h1 & m2



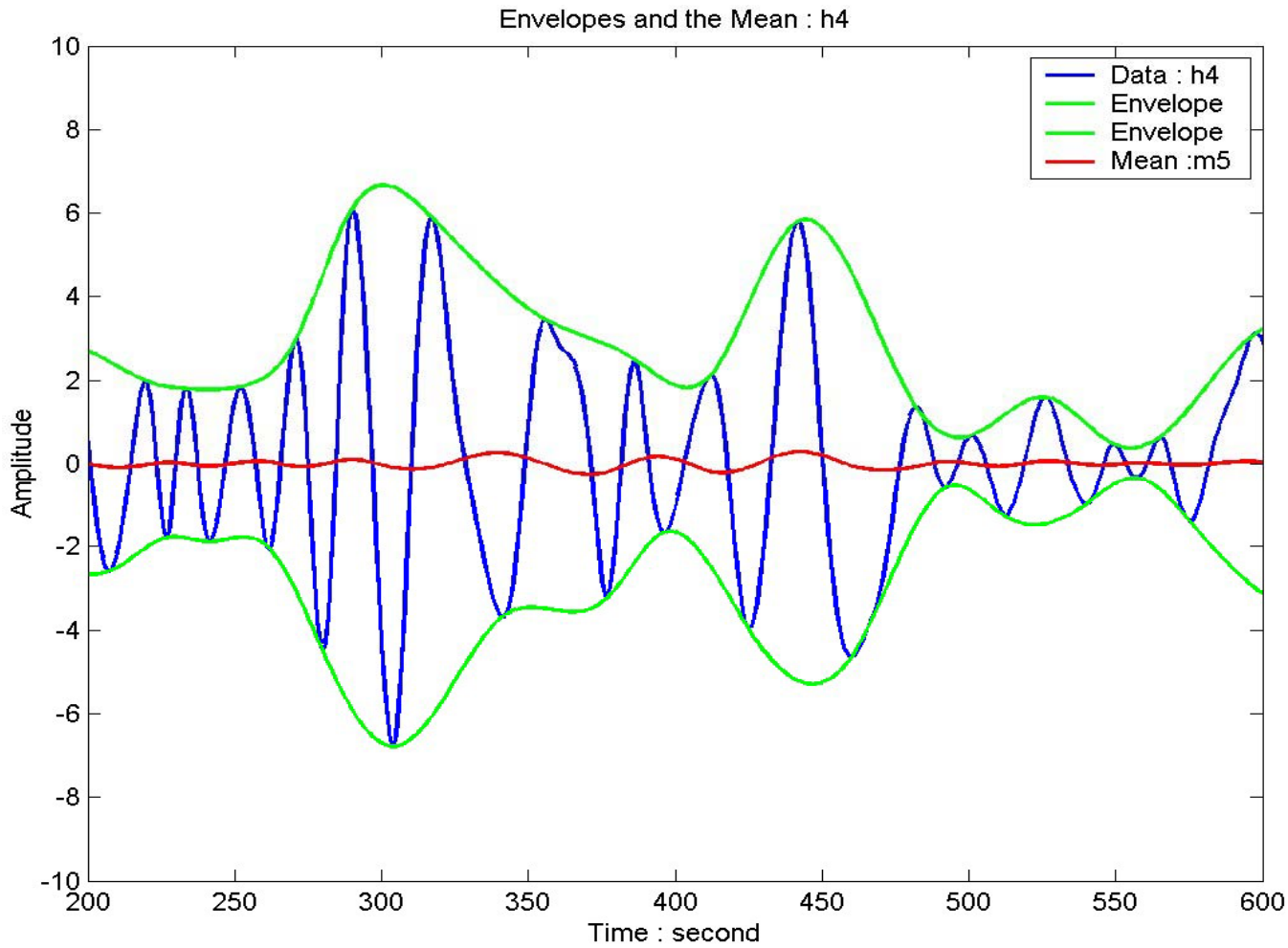
Empirical Mode Decomposition:

Methodology : h3 & m4



Empirical Mode Decomposition:

Methodology : h4 & m5



Empirical Mode Decomposition

Sifting : to get one IMF component

$$x(t) - m_1 = h_1 ,$$

$$h_1 - m_2 = h_2 ,$$

$$\cdots\cdots$$

$$\cdots\cdots$$

$$h_{k-1} - m_k = h_k .$$

$$\Rightarrow \quad h_k = c_1 .$$

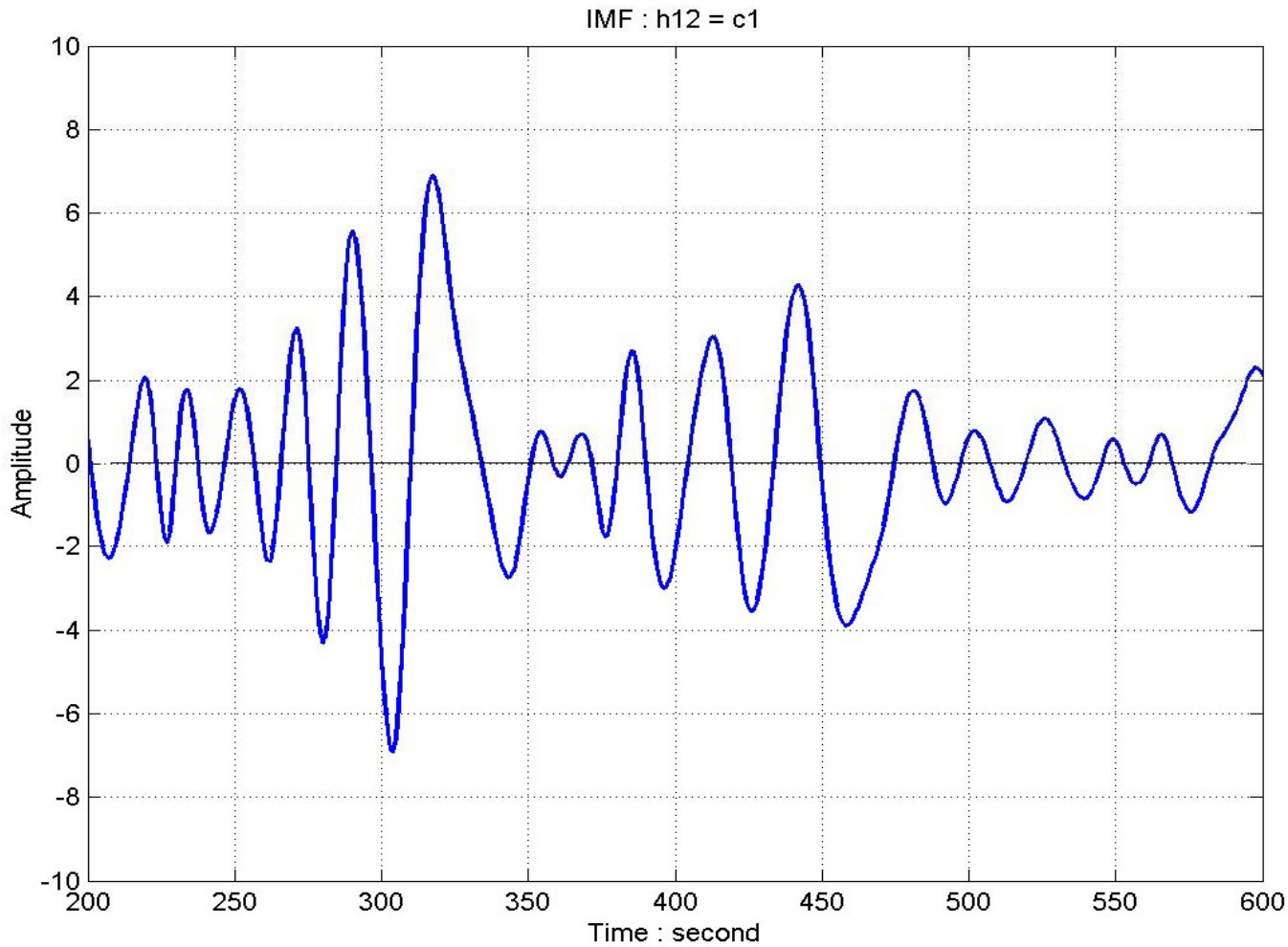
Two Stoppage Criteria : S and SD

A. The S number : S is defined as the consecutive number of siftings, in which the numbers of zero-crossing and extrema are the same for these S siftings.

B. SD is small than a pre-set value, where

$$SD = \sum_{t=0}^{T} \frac{\left| h_{k-1}(t) - h_k(t) \right|^2}{h_{k-1}^{\ 2}(t)} .$$

Empirical Mode Decomposition:
Methodology : IMF c1



Definition of the Intrinsic Mode Function (IMF)

Any function having the same numbers of zero – crossings and extrema, and also having symmetric envelopes defined by local maxima and minima respectively is defined as an Intrinsic Mode Function (IMF).

All IMF enjoys good Hilbert Transform :

$$\Rightarrow\Rightarrow \quad c(t) = a(t)e^{i\theta(t)}$$

Empirical Mode Decomposition

Sifting : to get all the IMF components

$$x(t) - c_1 = r_1 \ ,$$

$$r_1 - c_2 = r_2 \ ,$$

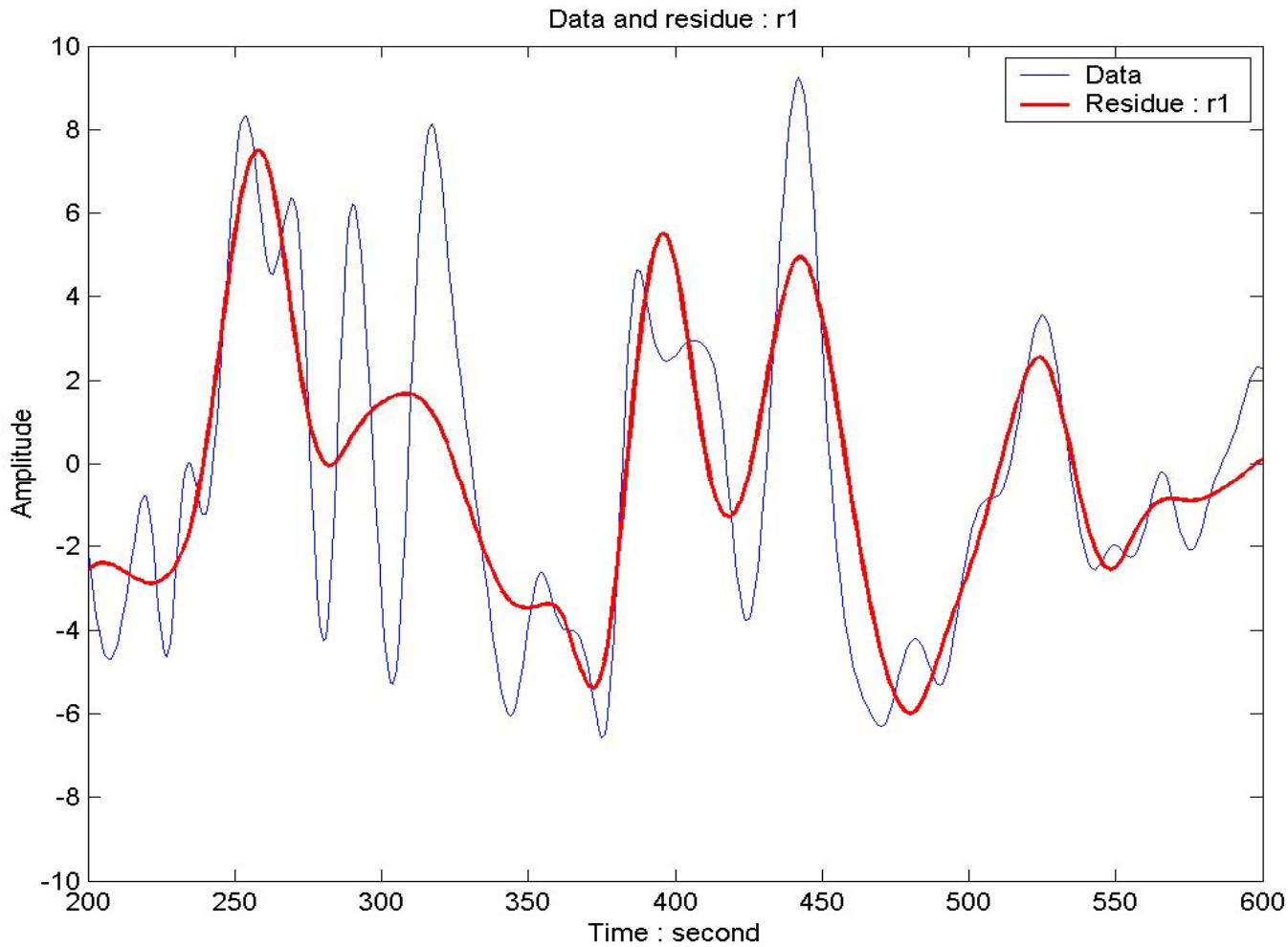
$$\cdots$$

$$r_{n-1} - c_n = r_n \ .$$

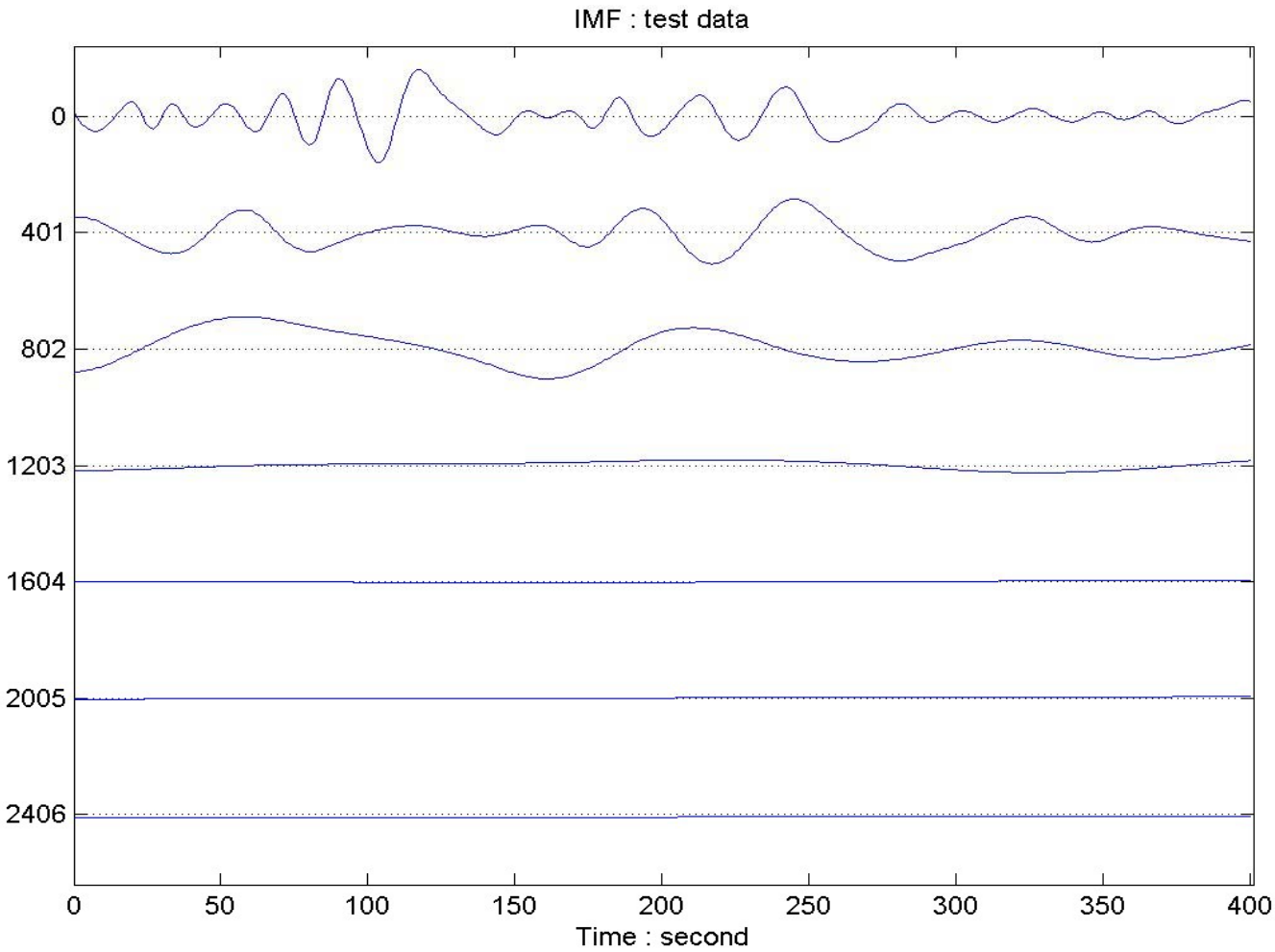
$$\Rightarrow \ x(t) - \sum_{j=1}^{n} c_j = r_n \ .$$

Empirical Mode Decomposition:

Methodology : data & r1



Empirical Mode Decomposition:
Methodology : IMFs



Definition of Instantaneous Frequency

The Fourier Transform of the Instrinsic Mode Funnction, c(t), gives

$$W(\omega) = \int_t a(t)\, e^{i(\theta - \omega t)}\, dt$$

By Stationary phase approximation we have

$$\frac{d\theta(t)}{dt} = \omega \ ,$$

This is defined as the Instantaneous Frequency.

Definitions of Frequency

1. Fourier Analysis :

$$F(\omega) = \sum_j \int_0^T x(t)\, e^{i\omega_j t}\, dt \ .$$

2. Wavelet Analysis

3. Wigner – Ville Analysis

4. Dynamic System through Hamiltonian :

$$H(p,q,t) \quad \text{and} \quad A(t) = \int_0^{TC} p\, dq \ ;$$

$$\Rightarrow \quad \omega = \frac{\partial H}{\partial A} .$$

5. Teager Energy Operator

6. Period between zero – crossings and extrema

7. HHT Analysis :

$$x(t) = \sum_j a(t)\, e^{i\theta_j(t)} \quad \Rightarrow \quad \omega_j = \frac{d\theta_j}{dt} .$$

Comparison between FFT and HHT

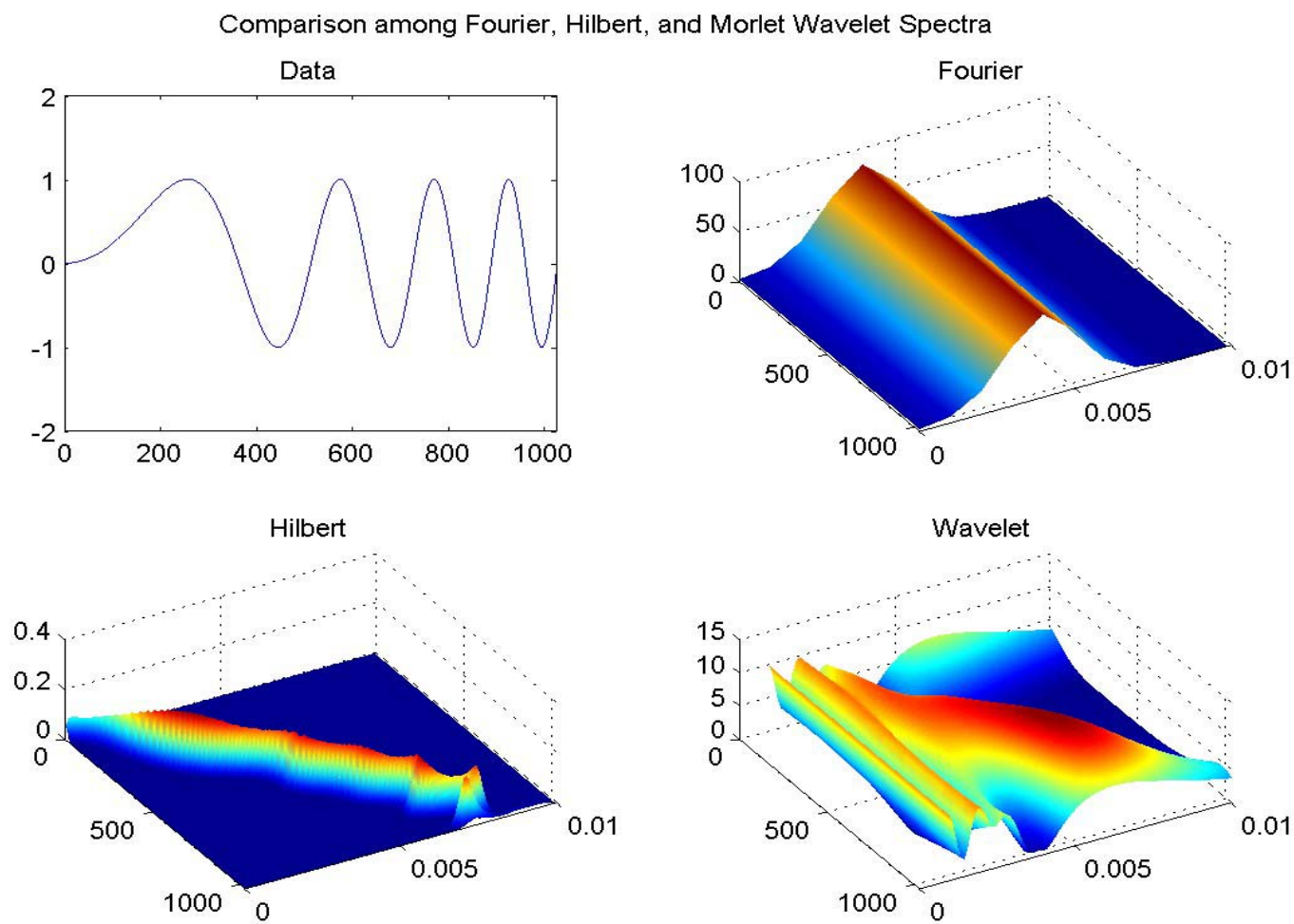
1. FFT :

$$x(t) = \Re \sum_j a_j e^{i\omega_j t} .$$

2. HHT :

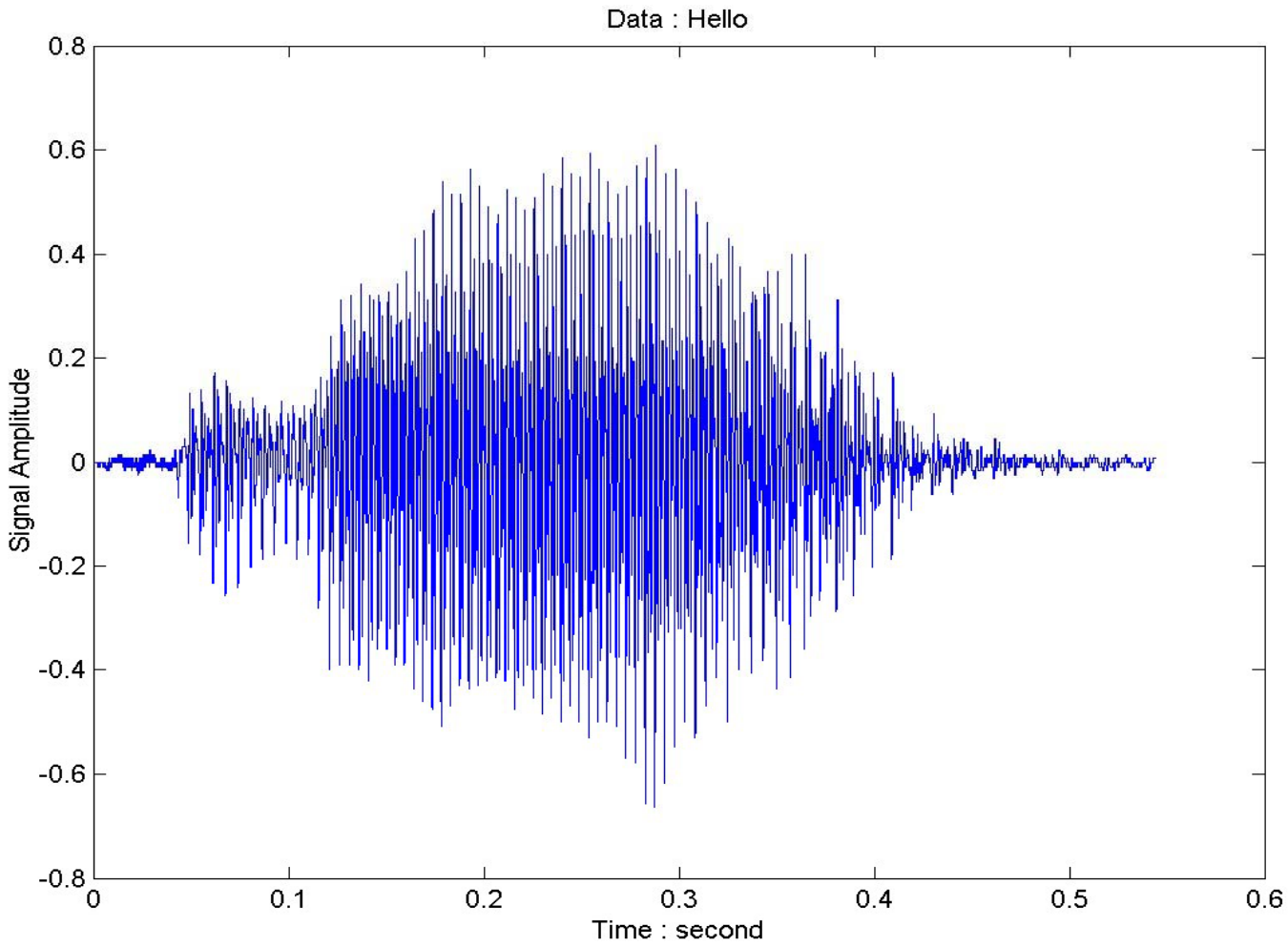
$$x(t) = \Re \sum_j a_j(t) e^{i\int_t \omega_j(\tau)d\tau} .$$

Comparisons:
Fourier, Hilbert & Wavelet

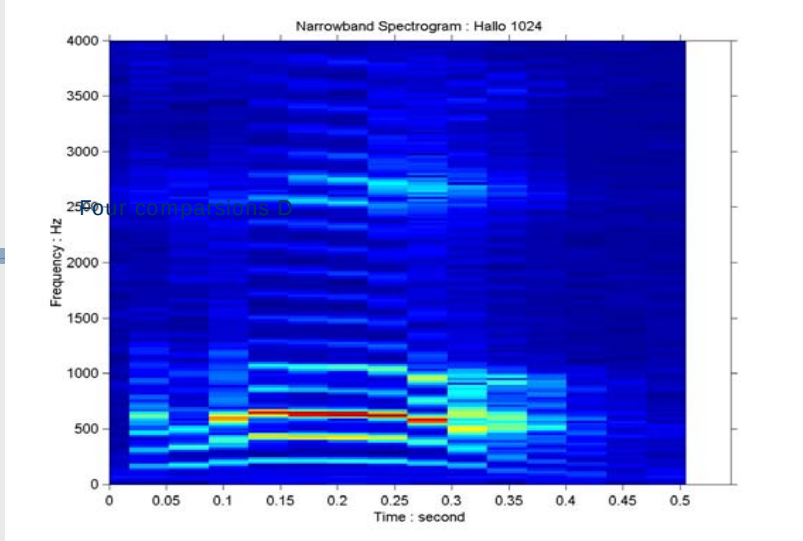


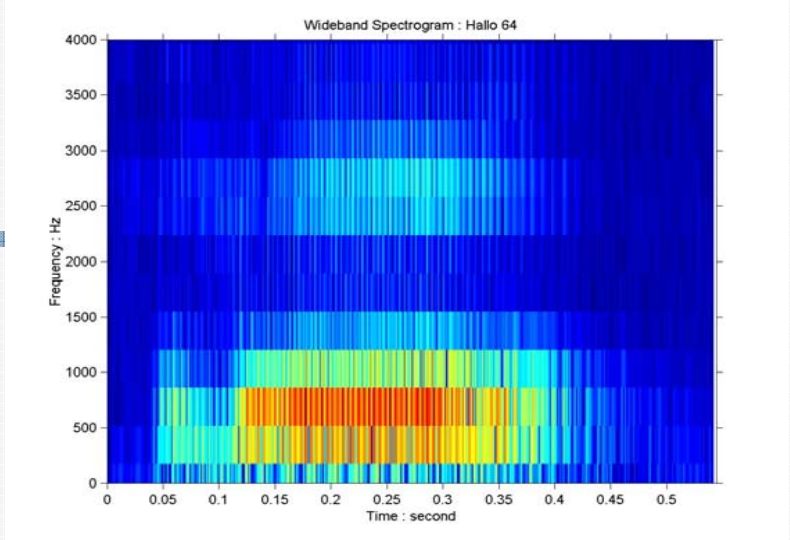
Speech Analysis

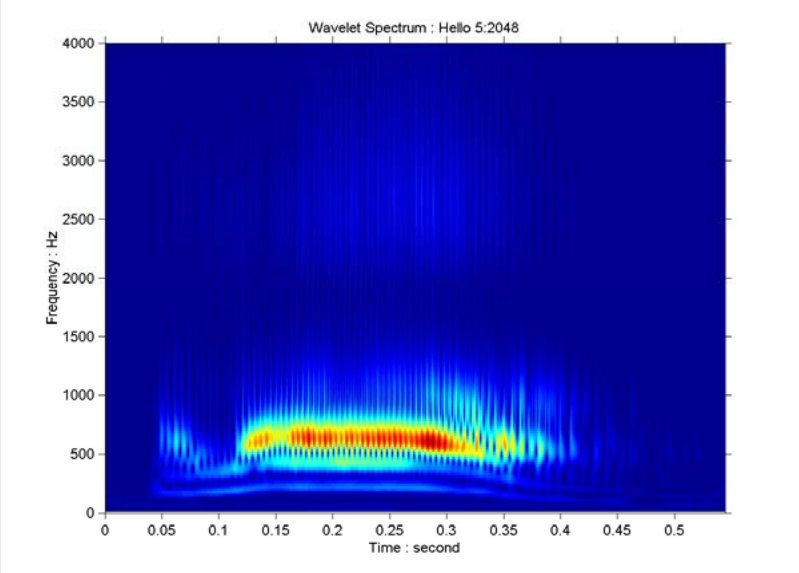
Hello : Data

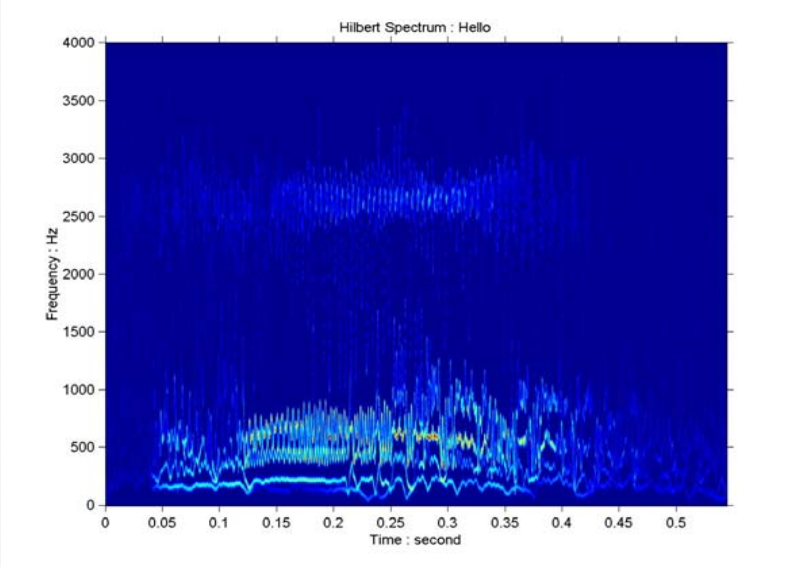


Four comparisons D



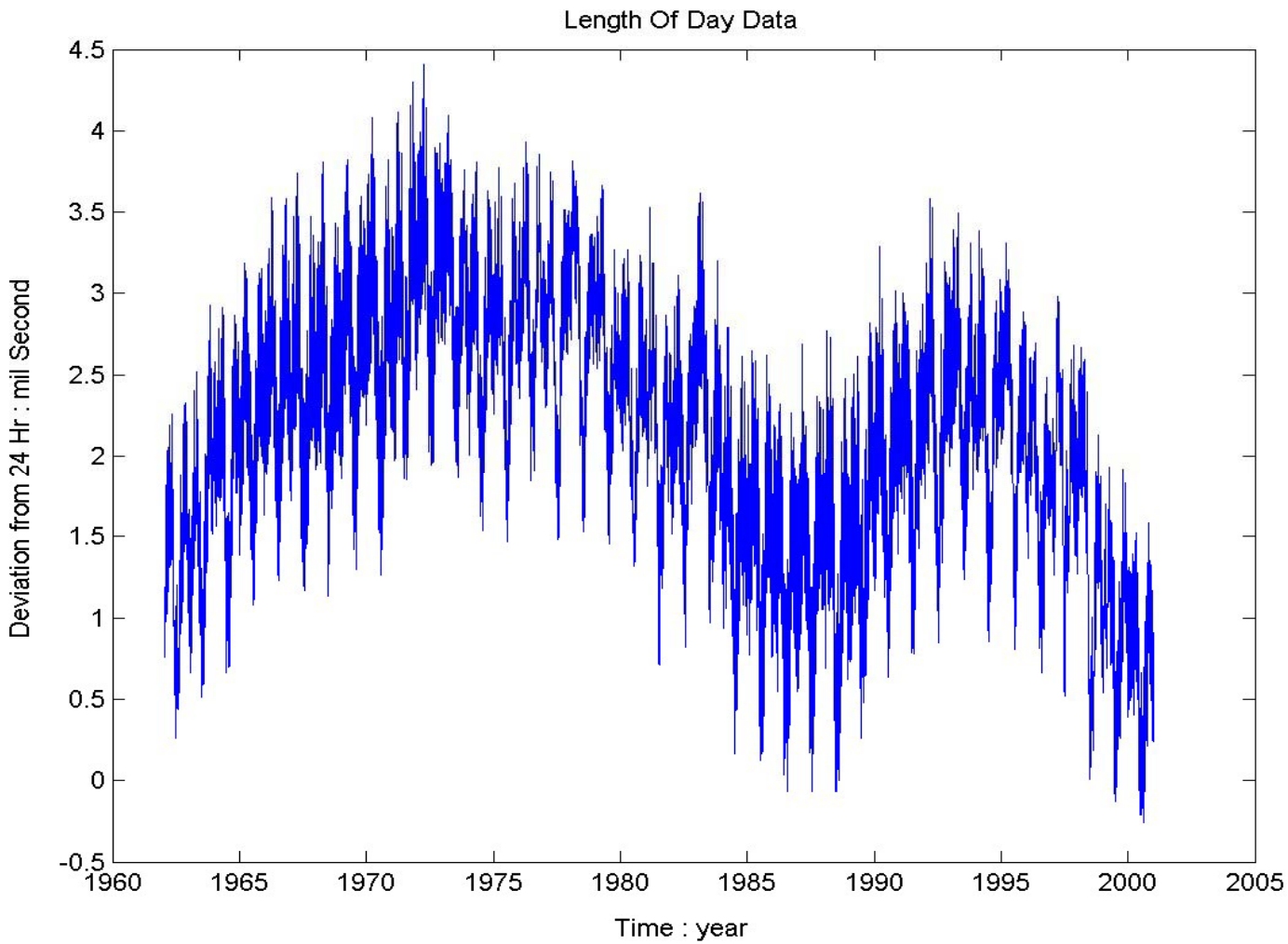




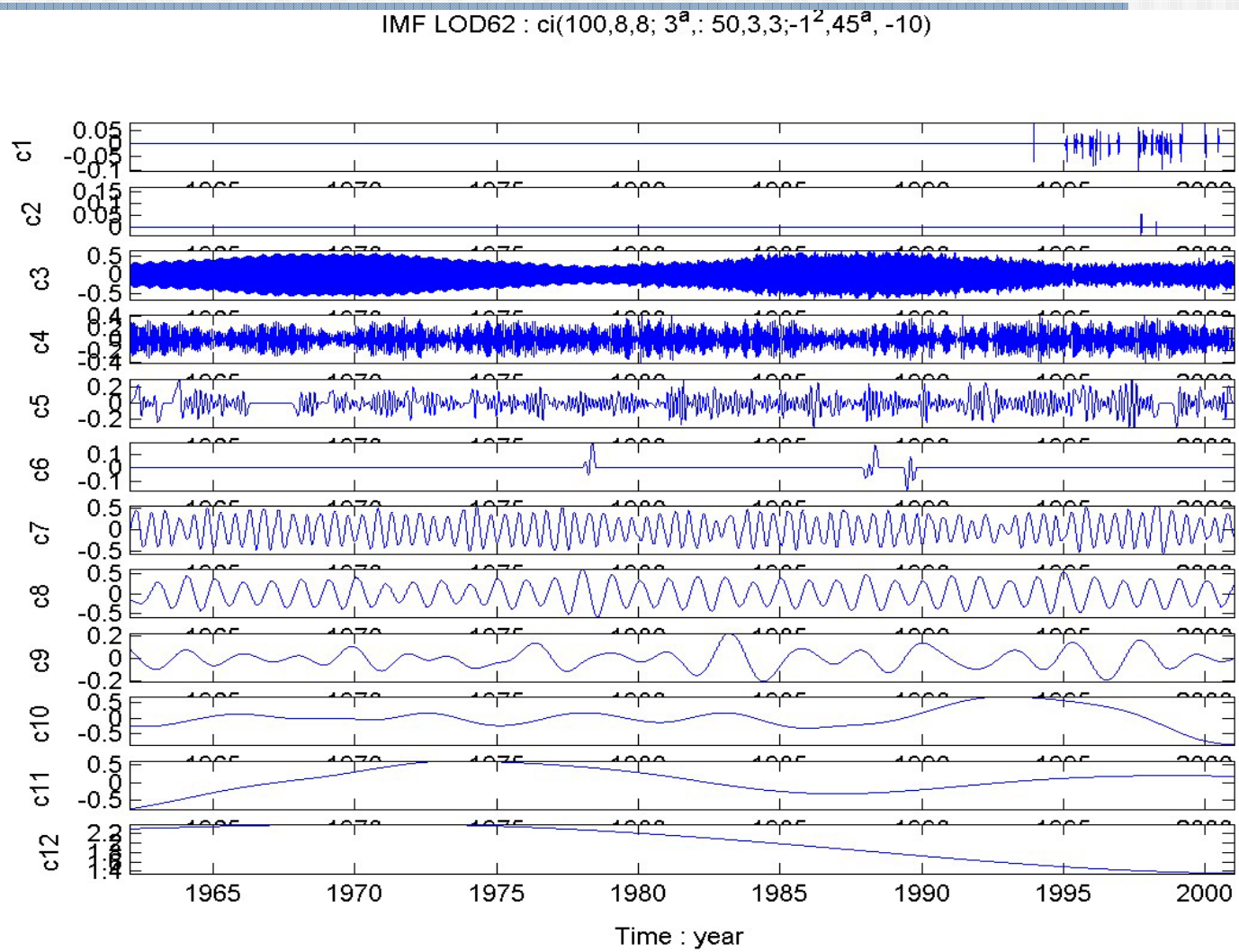


An Example of Sifting

Length Of Day Data



LOD : IMF

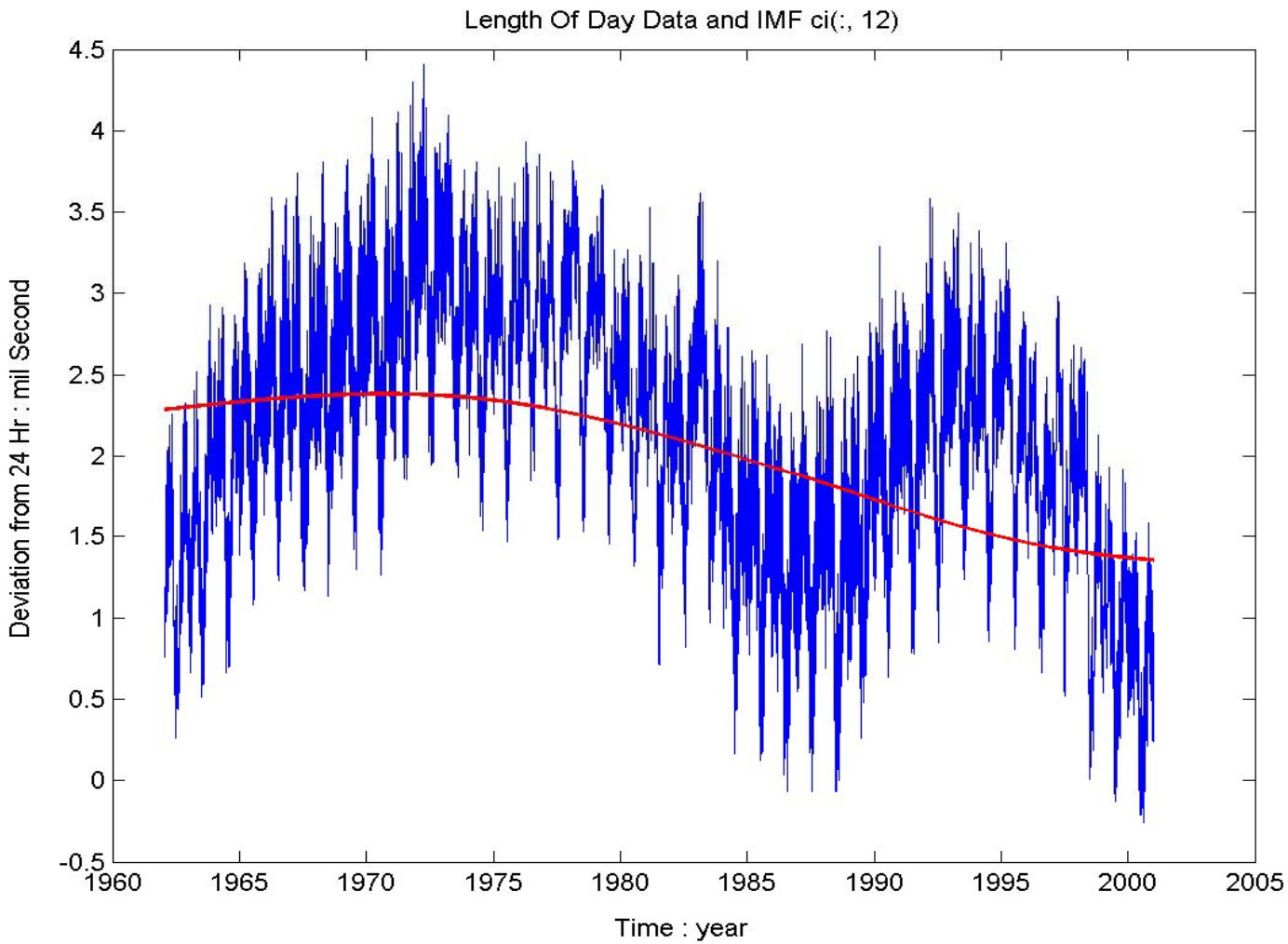

IMF LOD62 : ci(100,8,8; 3a,: 50,3,3;-12,45a, -10)
c1
c2
c3
c4
c5
c6
c7
c8
c9
c10
c11
c12
1965
1970
1975
1980
1985
1990
1995
2000
Time : year

Orthogonality Check

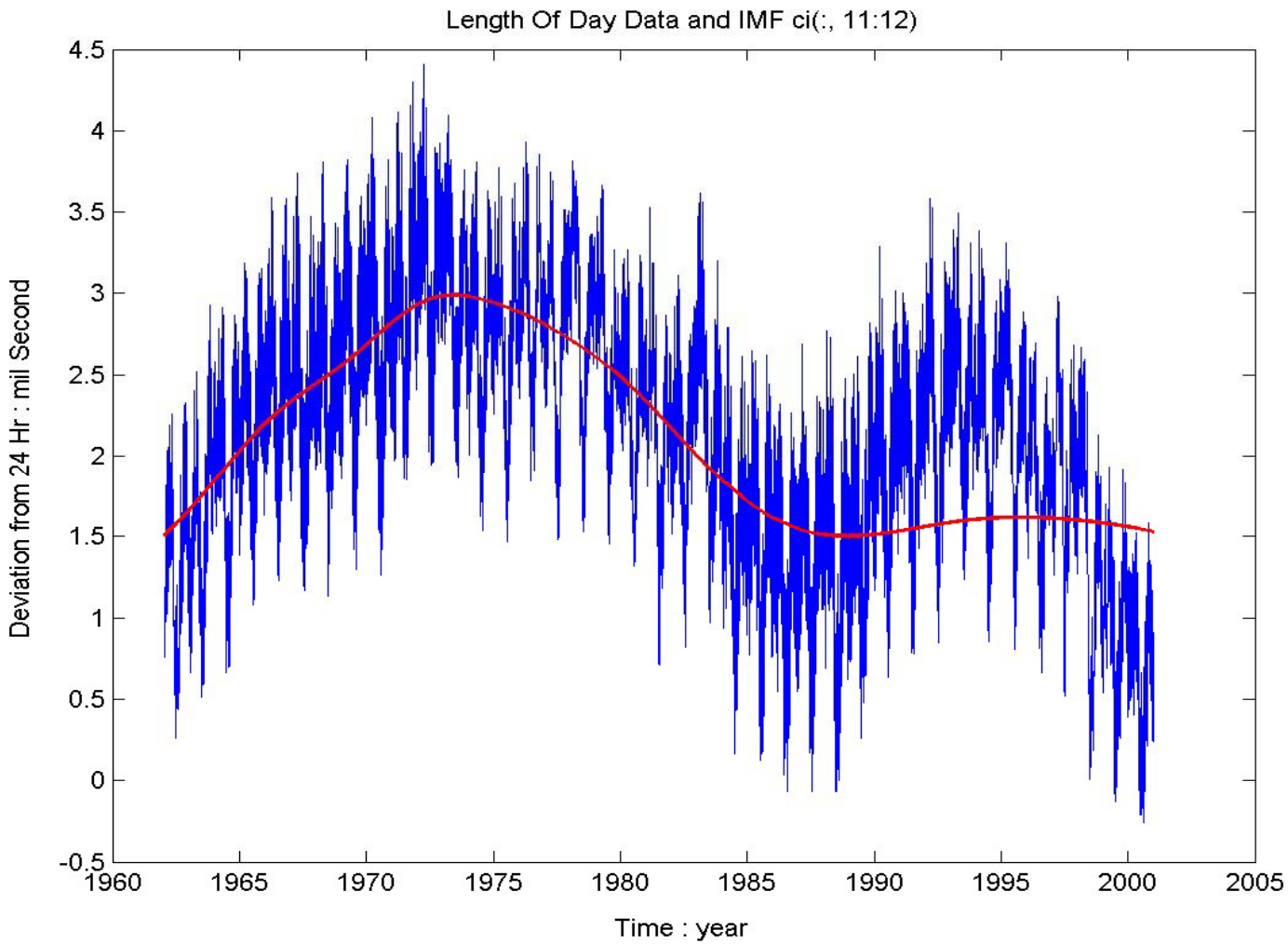
- Pair-wise %
- 0.0003
- 0.0001
- 0.0215
- 0.0117
- 0.0022
- 0.0031
- 0.0026
- 0.0083
- 0.0042
- 0.0369
- 0.0400

- Overall %
- 0.0452

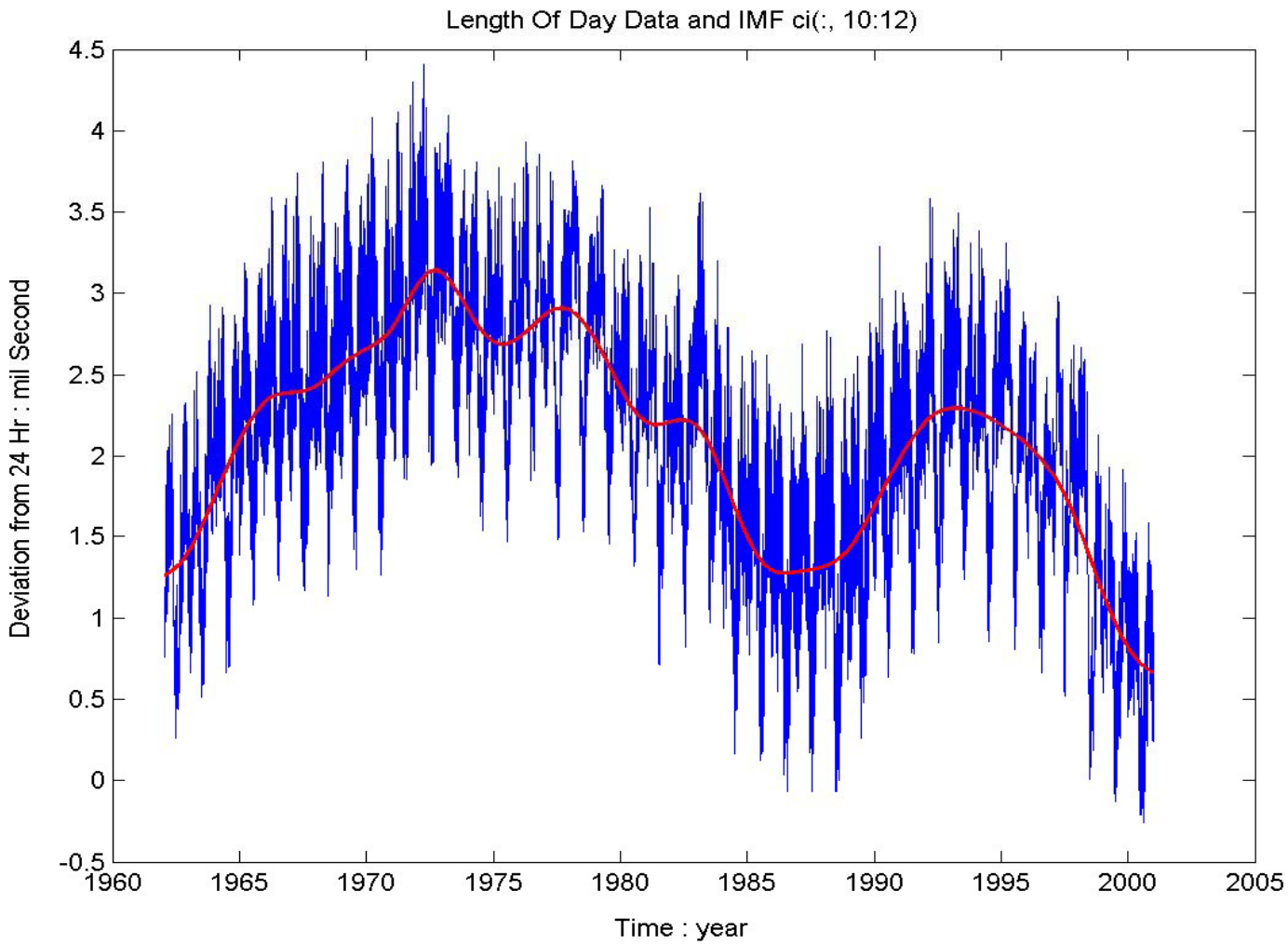
LOD : Data & c12


Length Of Day Data and IMF ci(:, 12)
Deviation from 24 Hr : mil Second
Time : year
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

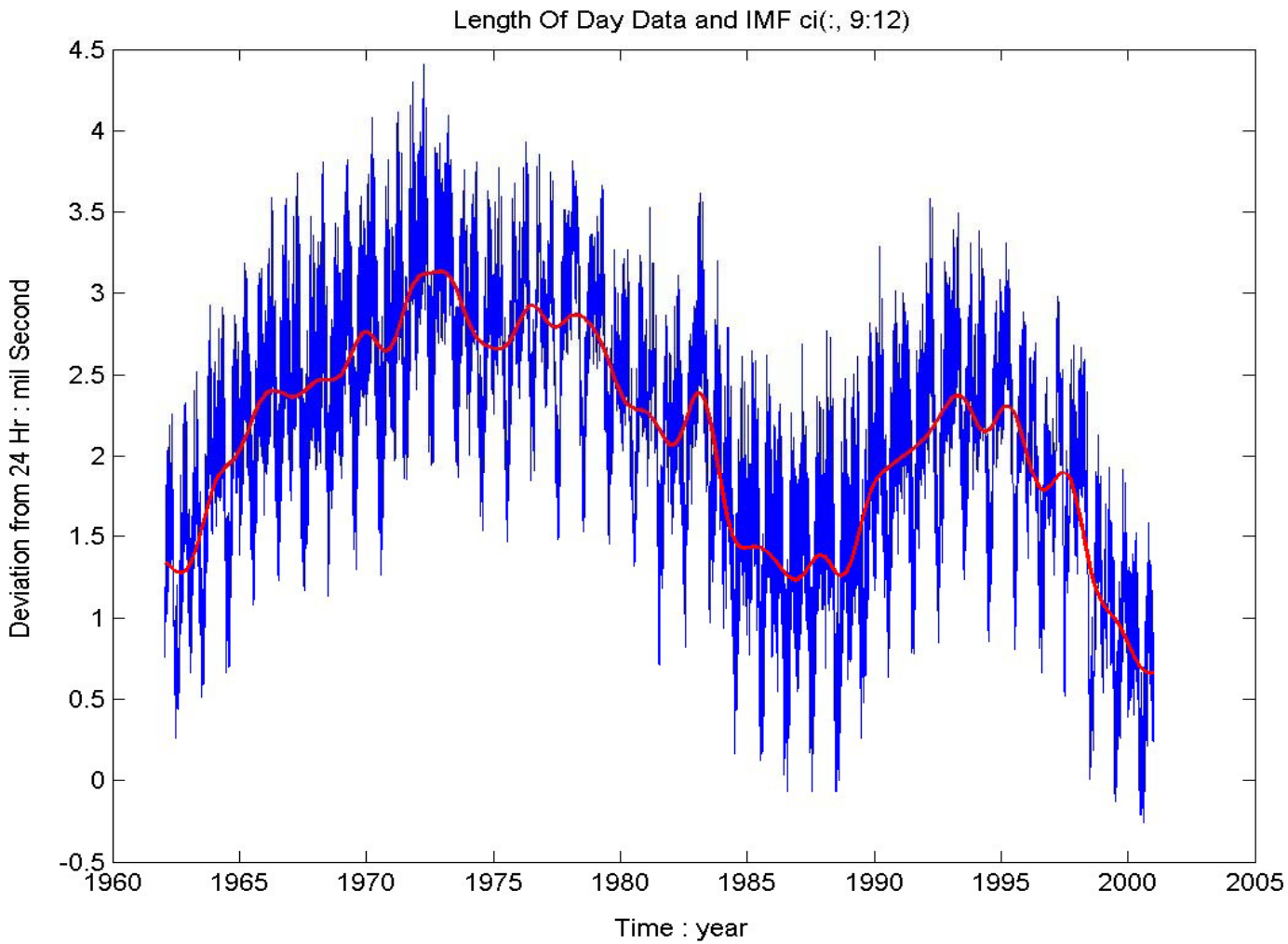
LOD : Data & Sum c11-12



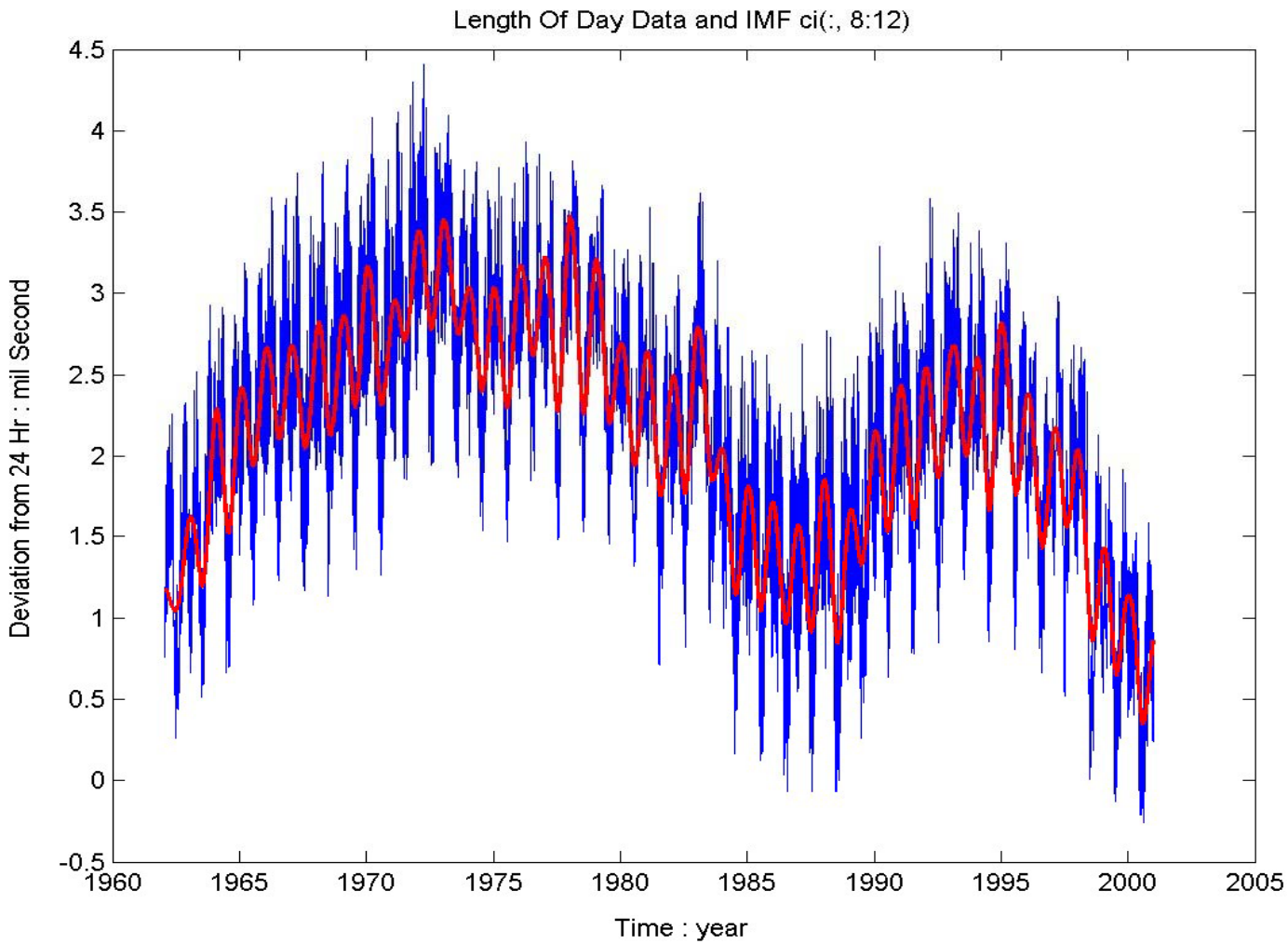
LOD : Data & sum c10-12



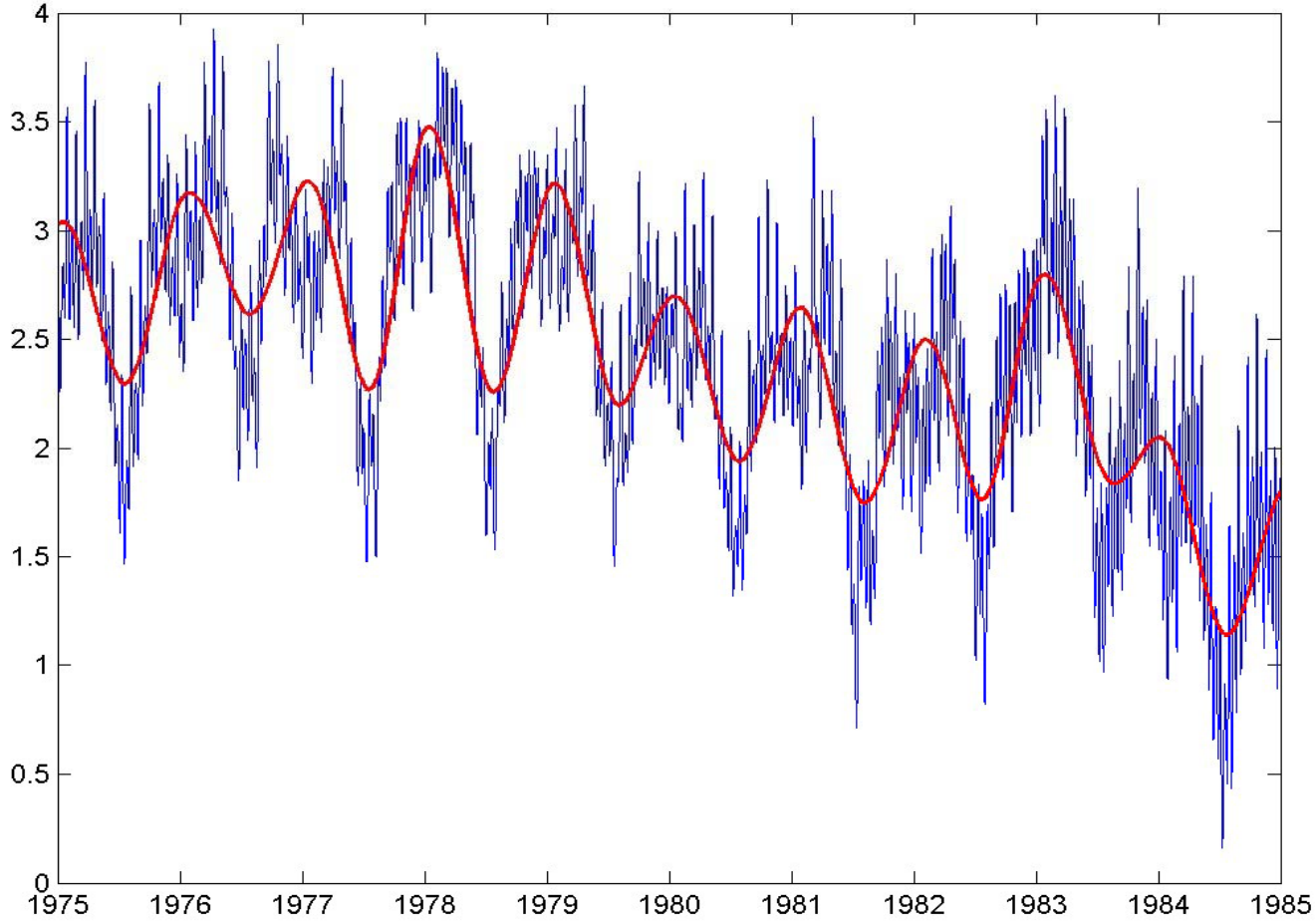
LOD : Data & c9 - 12



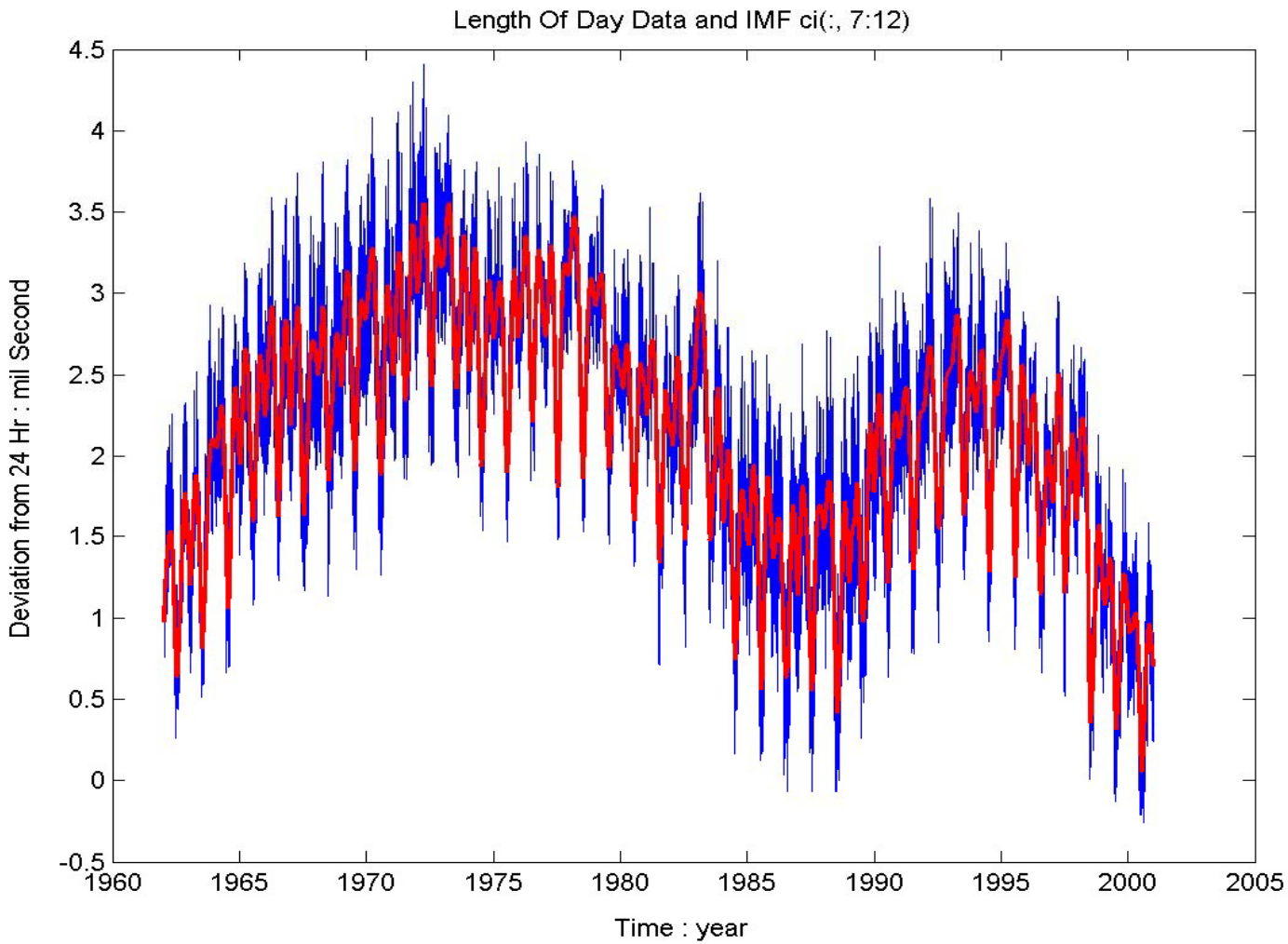
LOD : Data & c8 - 12



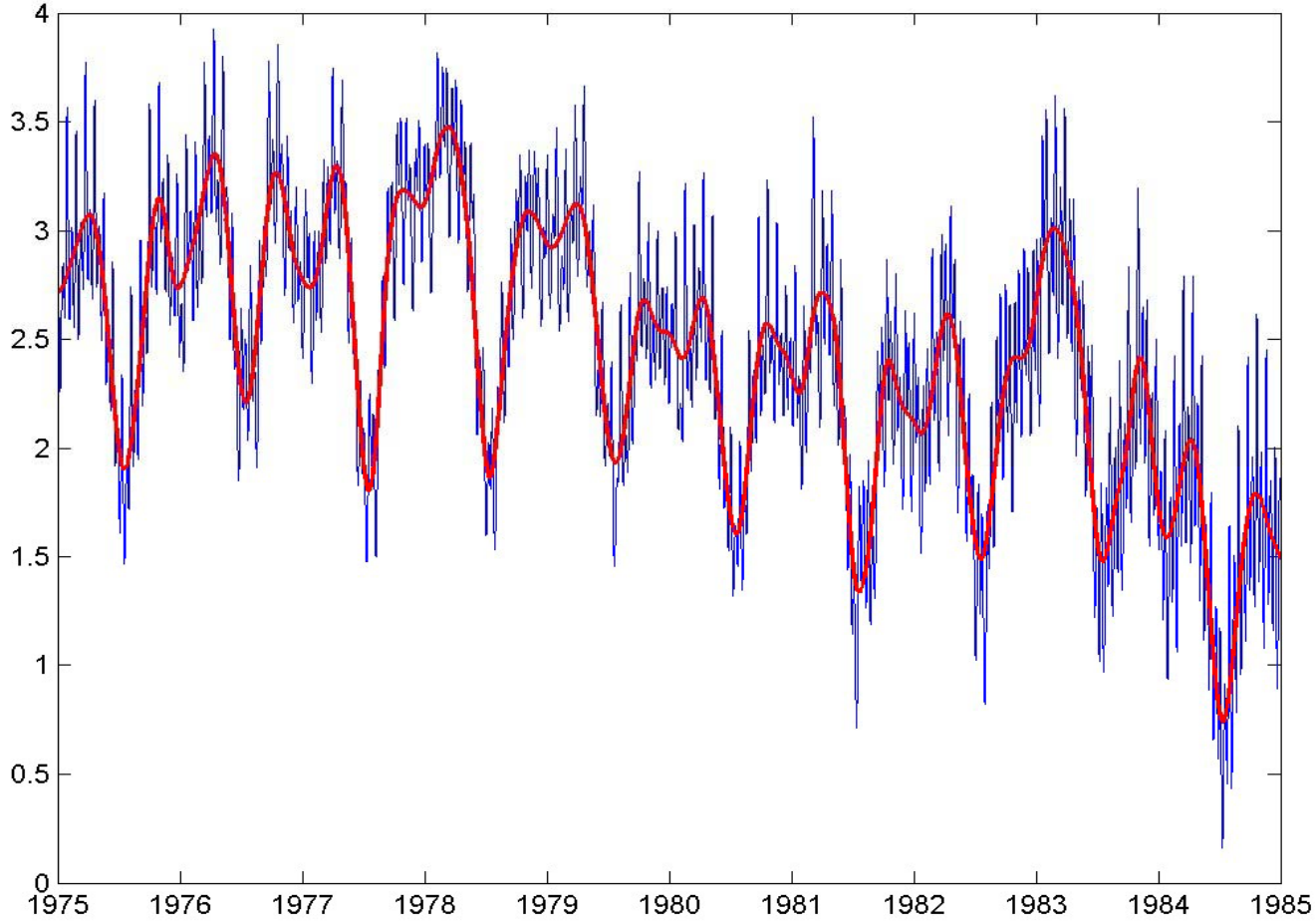
LOD : Detailed Data and Sum c8-c12



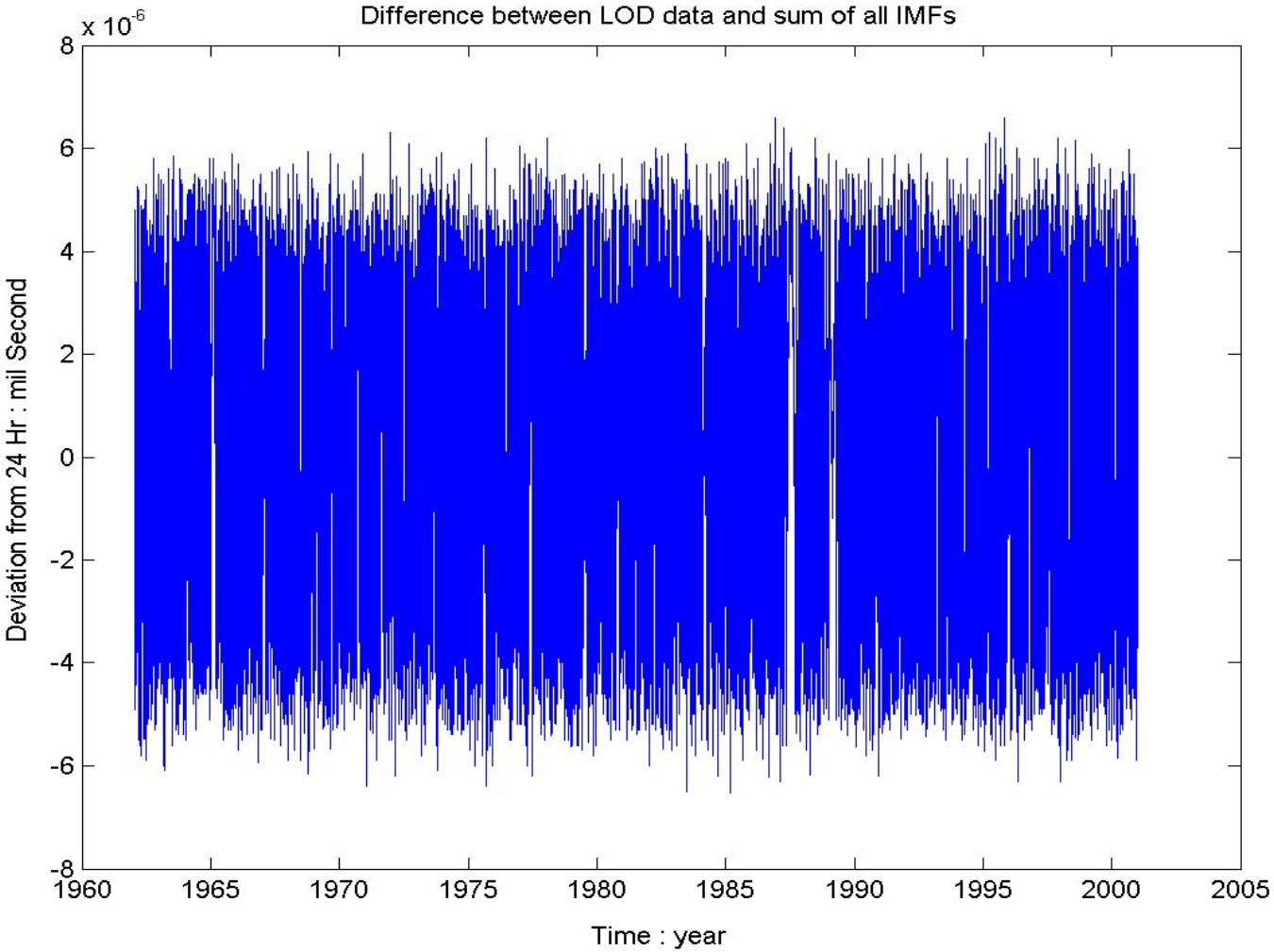
LOD : Data & c7 - 12



LOD : Detail Data and Sum IMF c7-c12

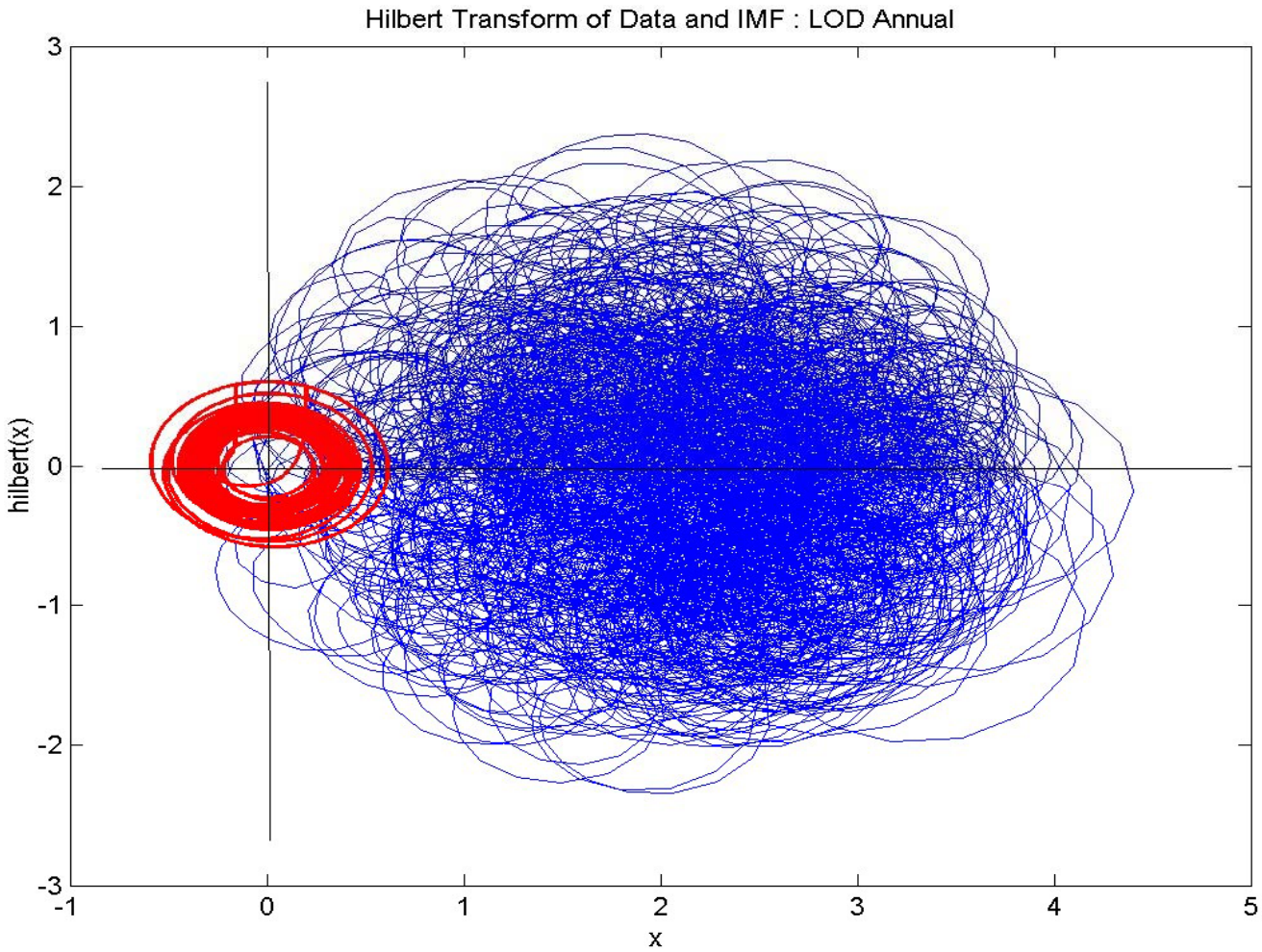


LOD : Difference Data – sum all IMFs

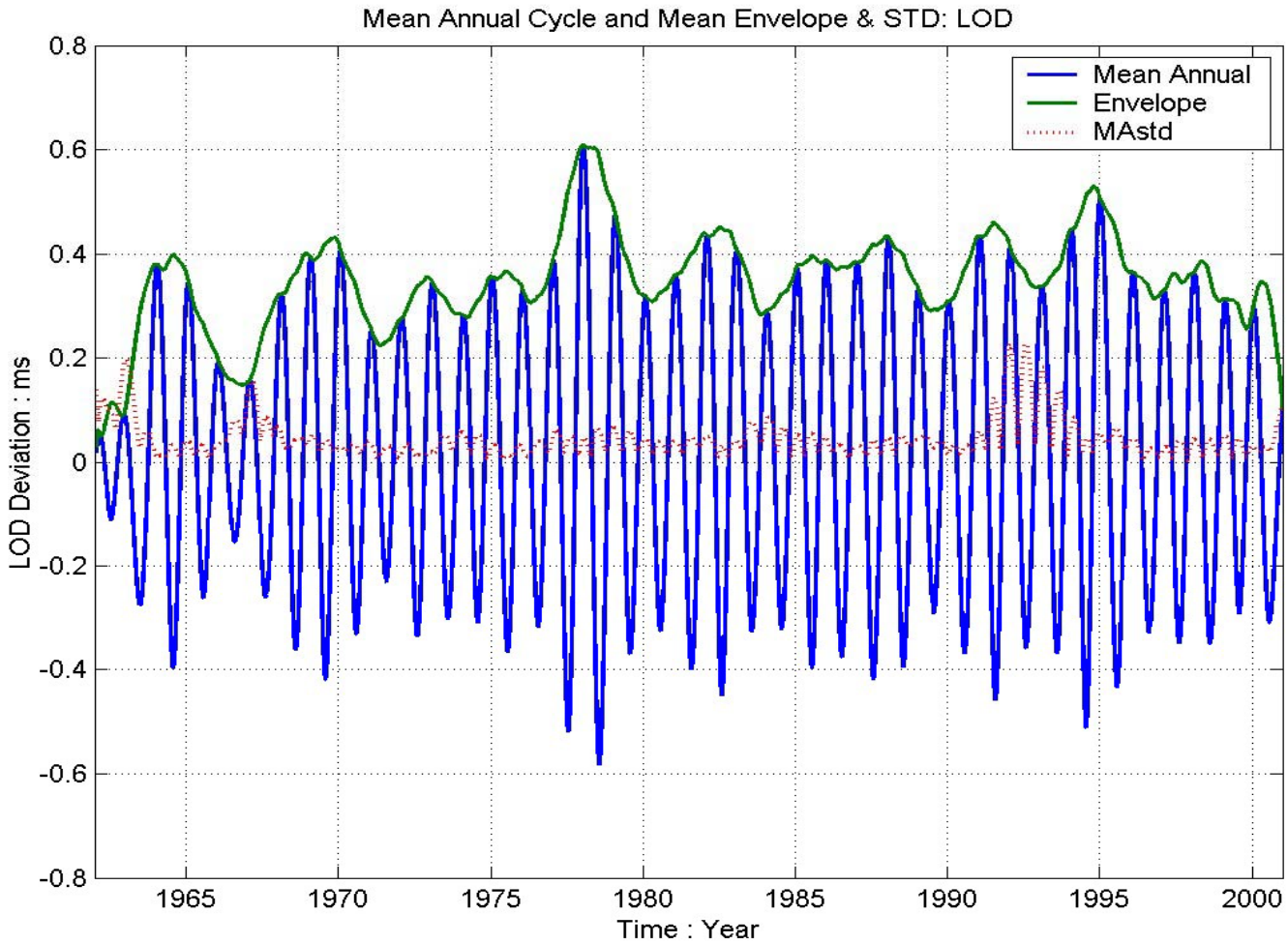


Traditional View

a la Hahn (1995) : Hilbert



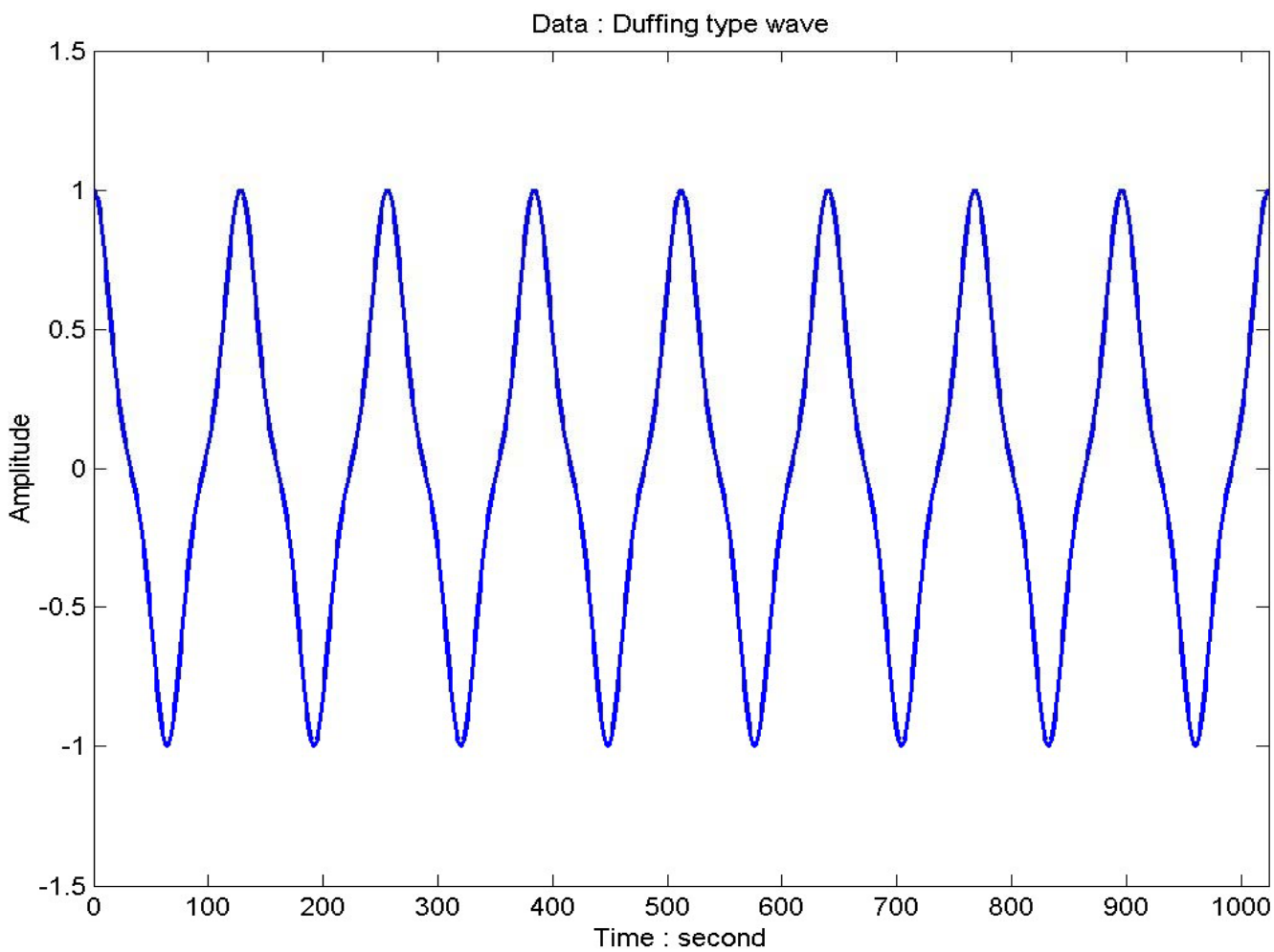
Mean Annual Cycle & Envelope: 9 CEI Cases



Hilbert's View on Nonlinear Data

Duffing Type Wave

Data: $x = \cos(wt + 0.3 \sin 2wt)$



Duffing Type Wave

Perturbation Expansion

For $\varepsilon = 1$, *we can have*

$$x(t) = \cos\left(\omega t + \varepsilon \sin 2\omega t\right)$$

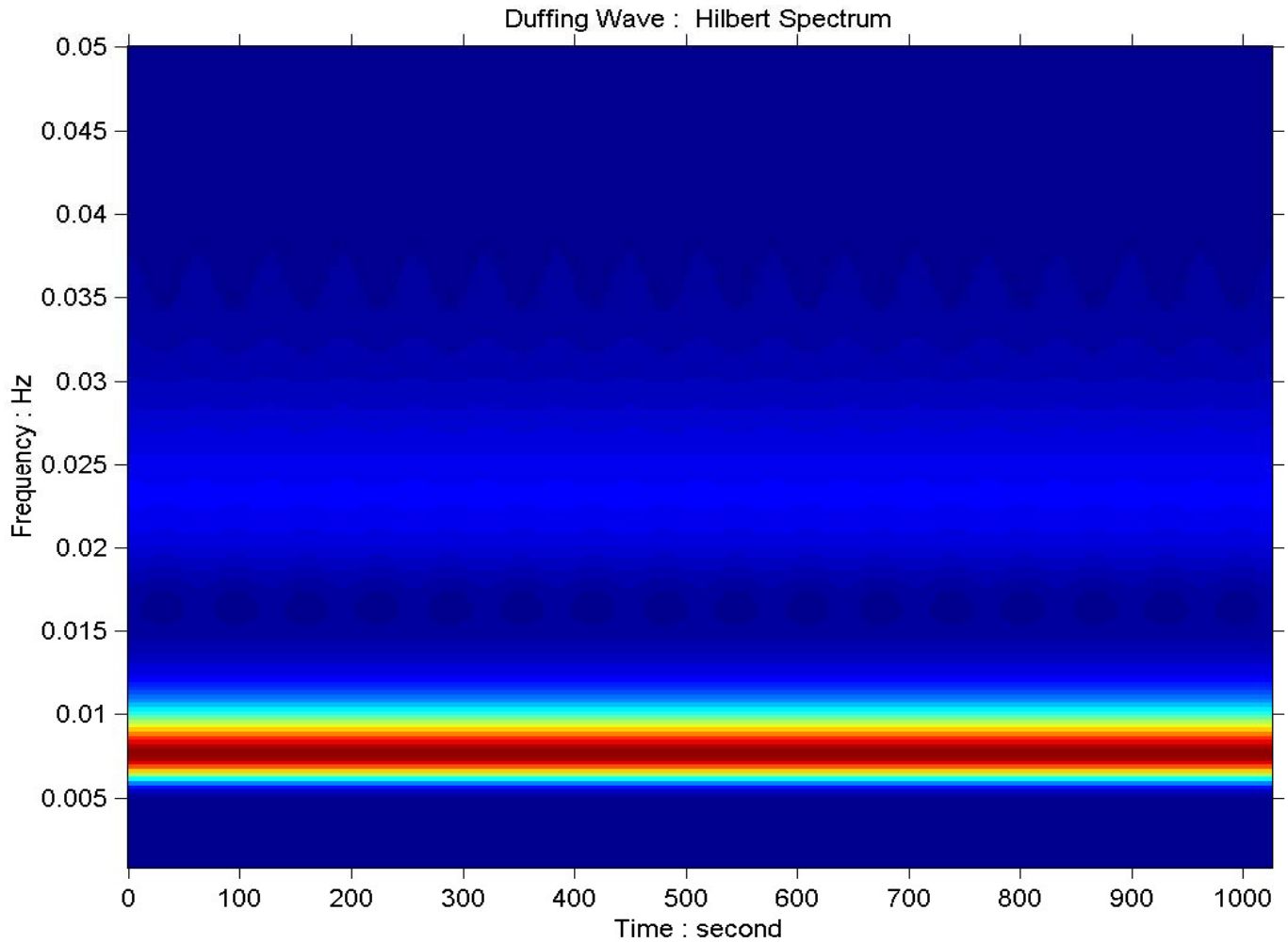
$$= \cos\omega t \cos\left(\varepsilon \sin 2\omega t\right) - \sin\omega t \sin\left(\varepsilon \sin 2\omega t\right)$$

$$= \cos\omega t - \varepsilon \sin\omega t \sin 2\omega t + \ldots.$$

$$= \left(1 - \frac{\varepsilon}{2}\right)\cos\omega t + \frac{\varepsilon}{2}\cos 3\omega t + \ldots.$$

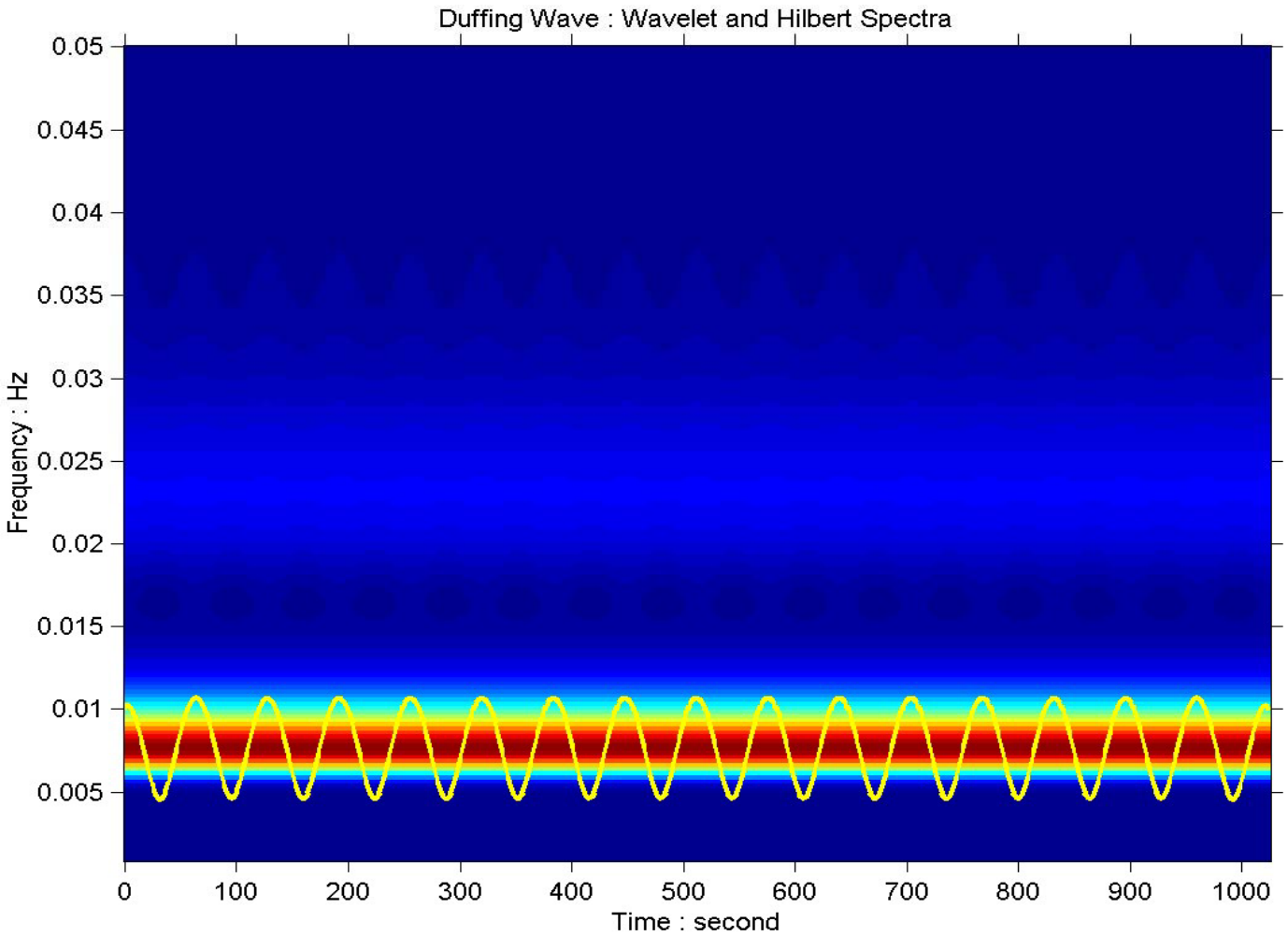
This is very similar to the solution of Duffing equation .

Duffing Type Wave
Wavelet Spectrum

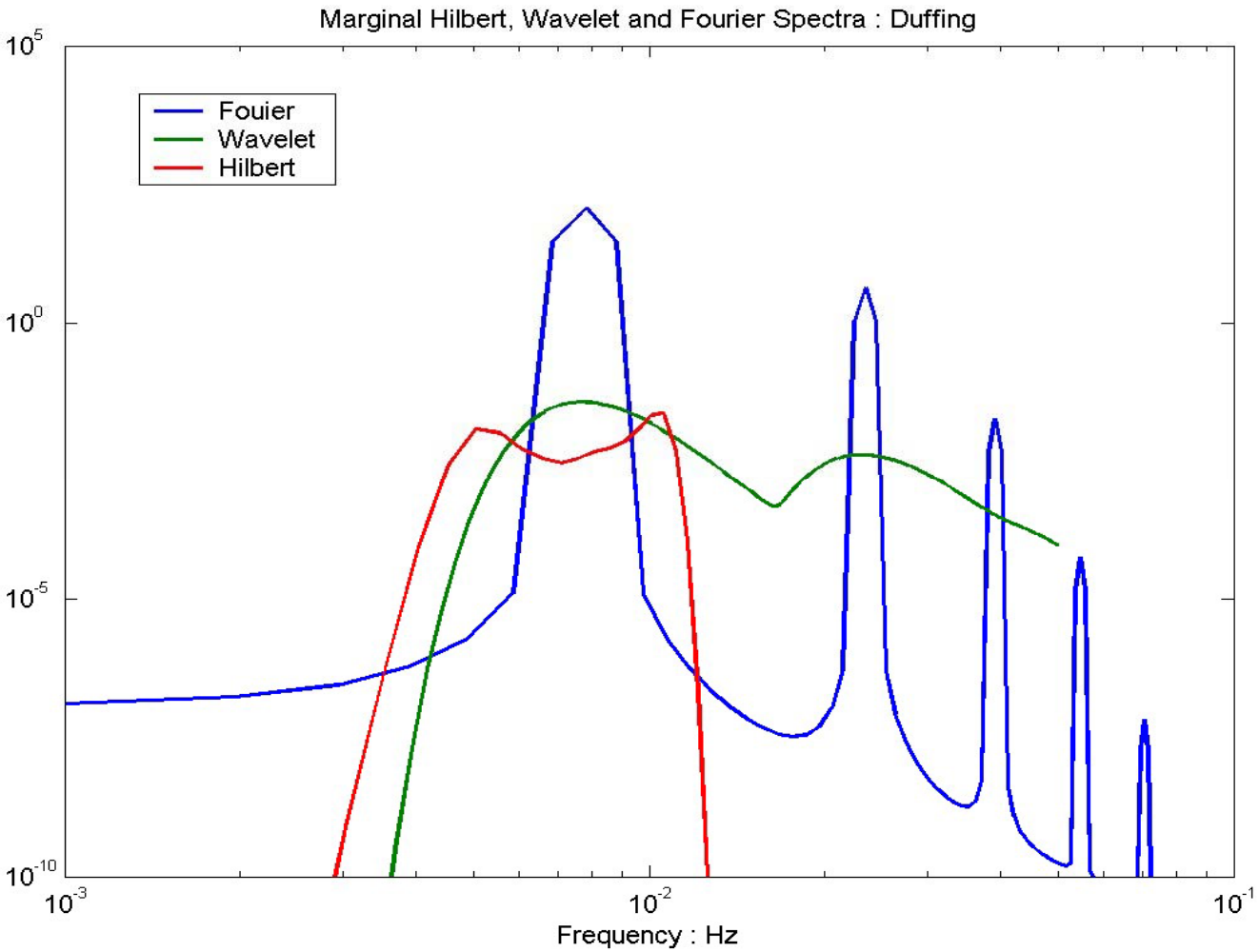


Duffing Type Wave

Hilbert Spectrum



Duffing Type Wave
Marginal Spectra



Duffing Equation

$$\frac{d^2 x}{dt^2} + x + \varepsilon x^3 = \gamma \cos \omega t \ .$$

Solved with ***ode23tb*** *for* $t = 0$ *to* 200 *with*

$\varepsilon = -1$

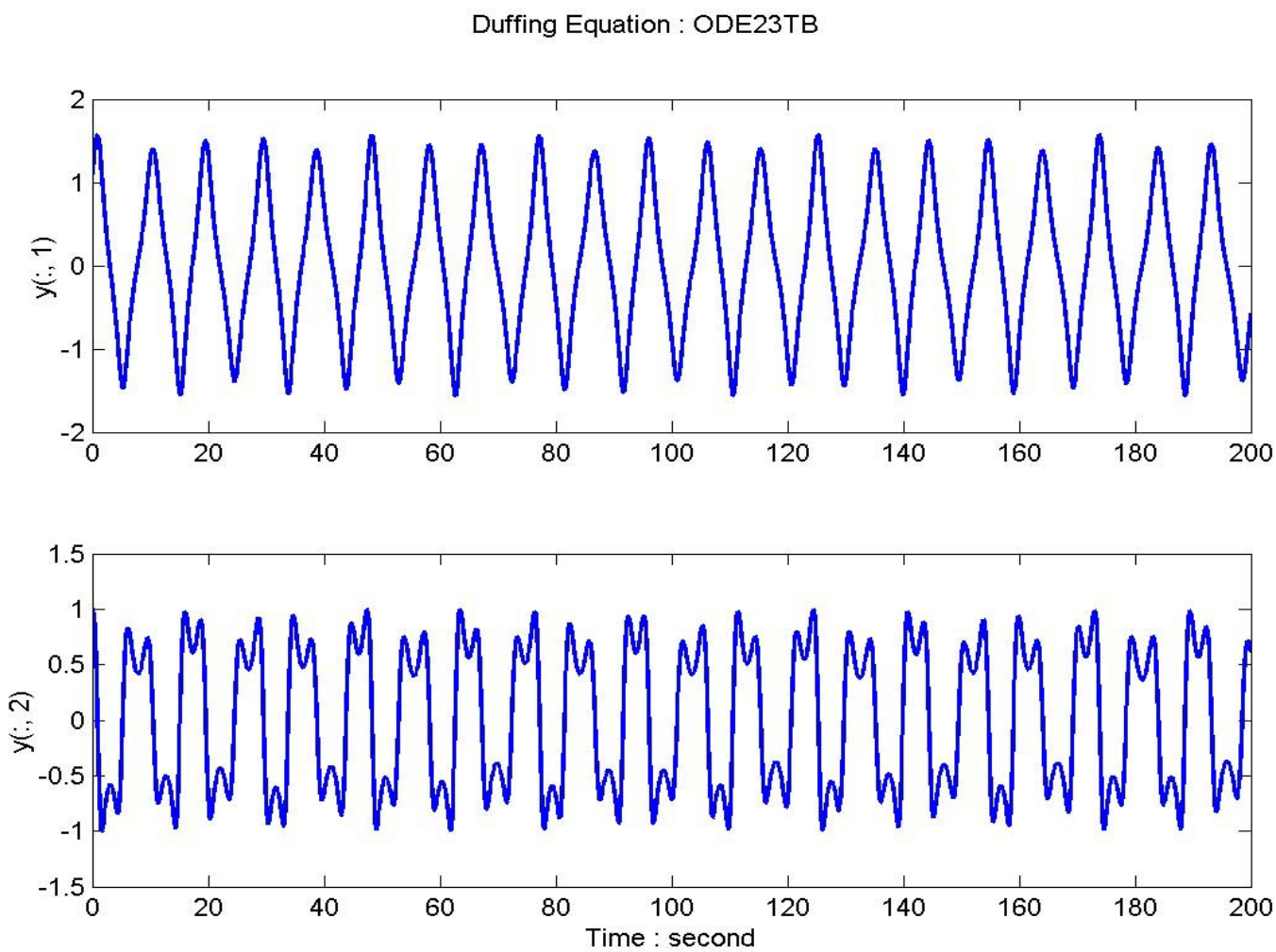
$\gamma = 0.1$

$\omega = 0.04\ Hz$

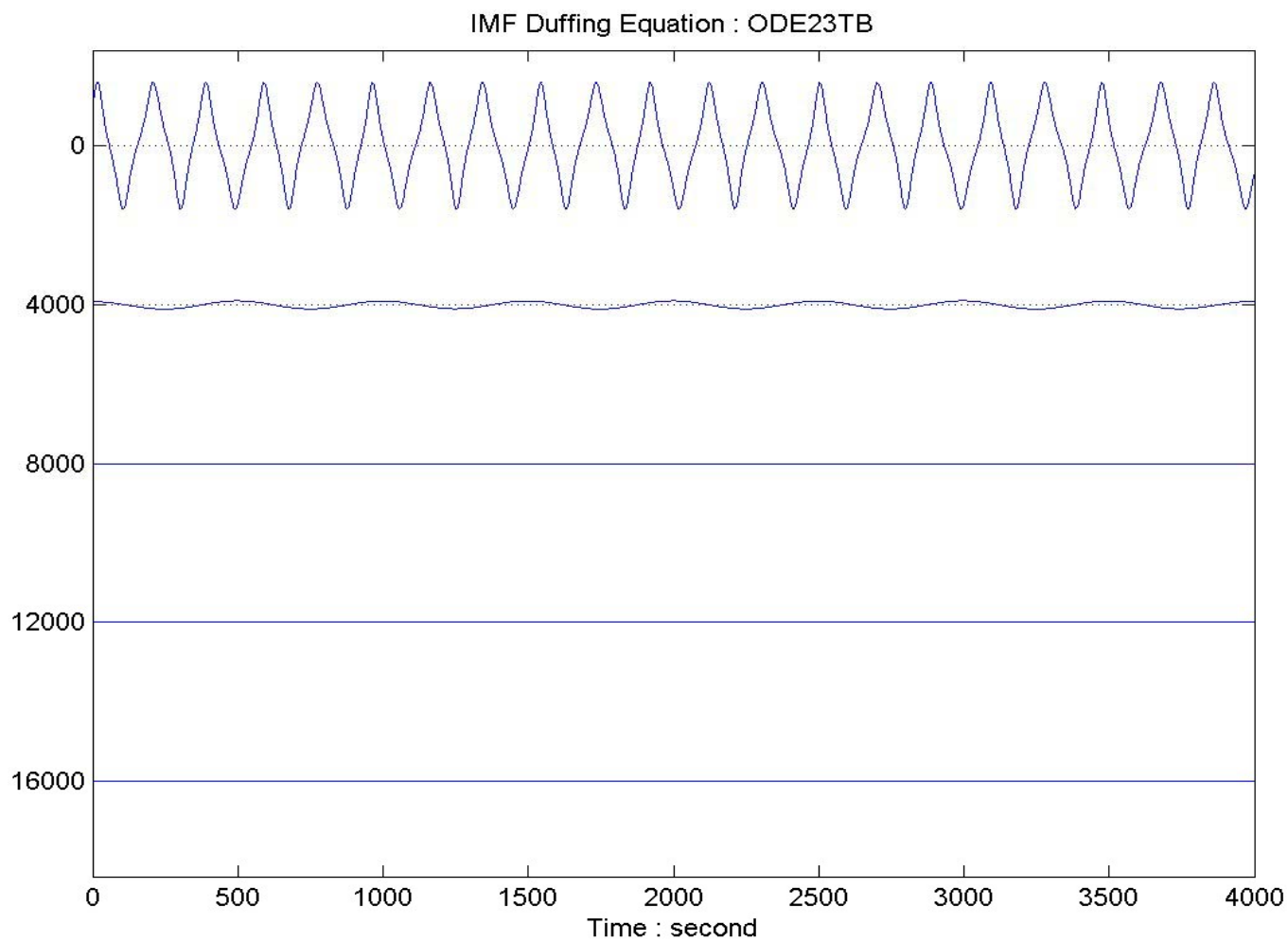
Initial condition :

$[x(o), x'(0)] = [1, 1]$

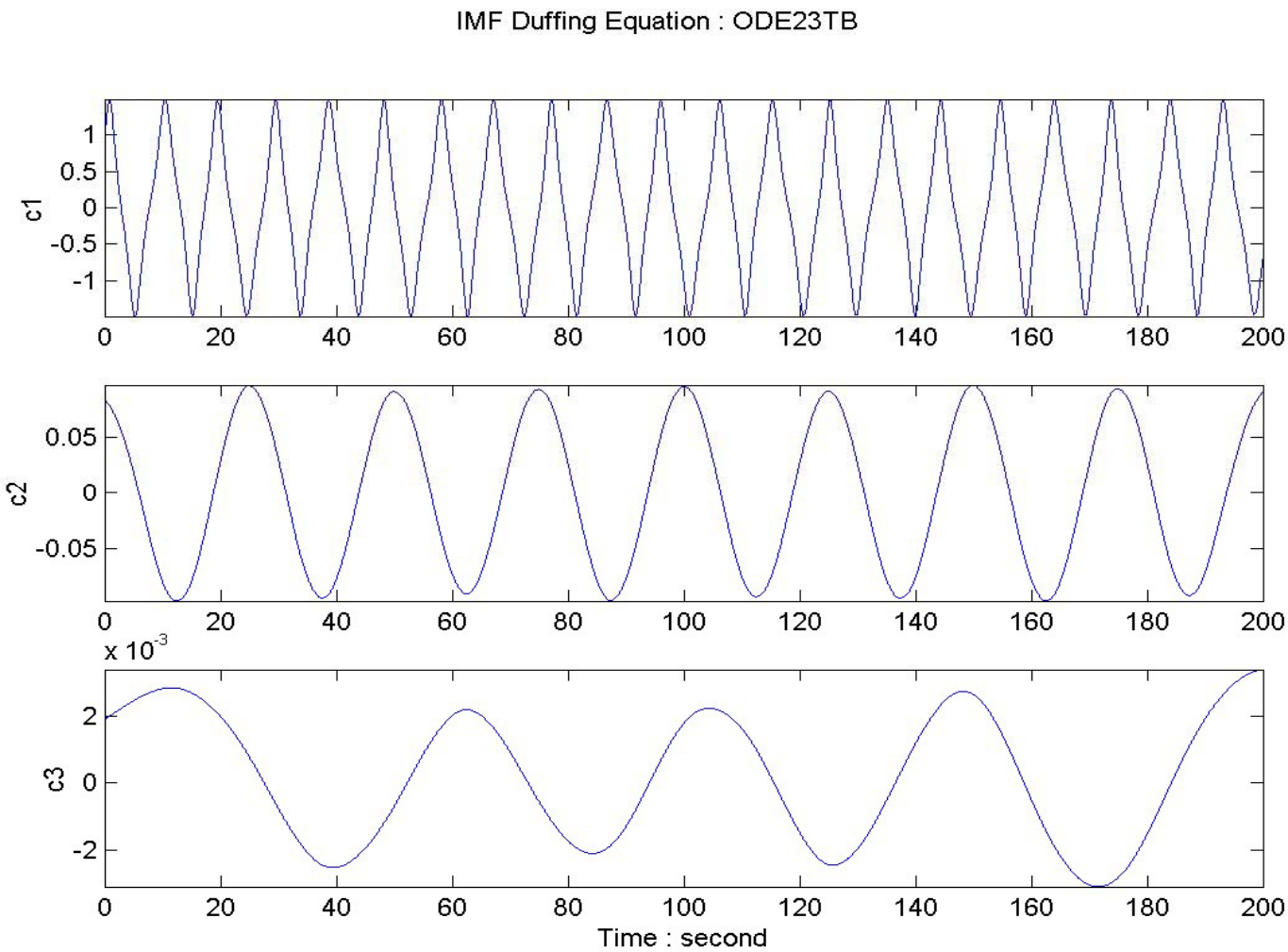
Duffing Equation : Data



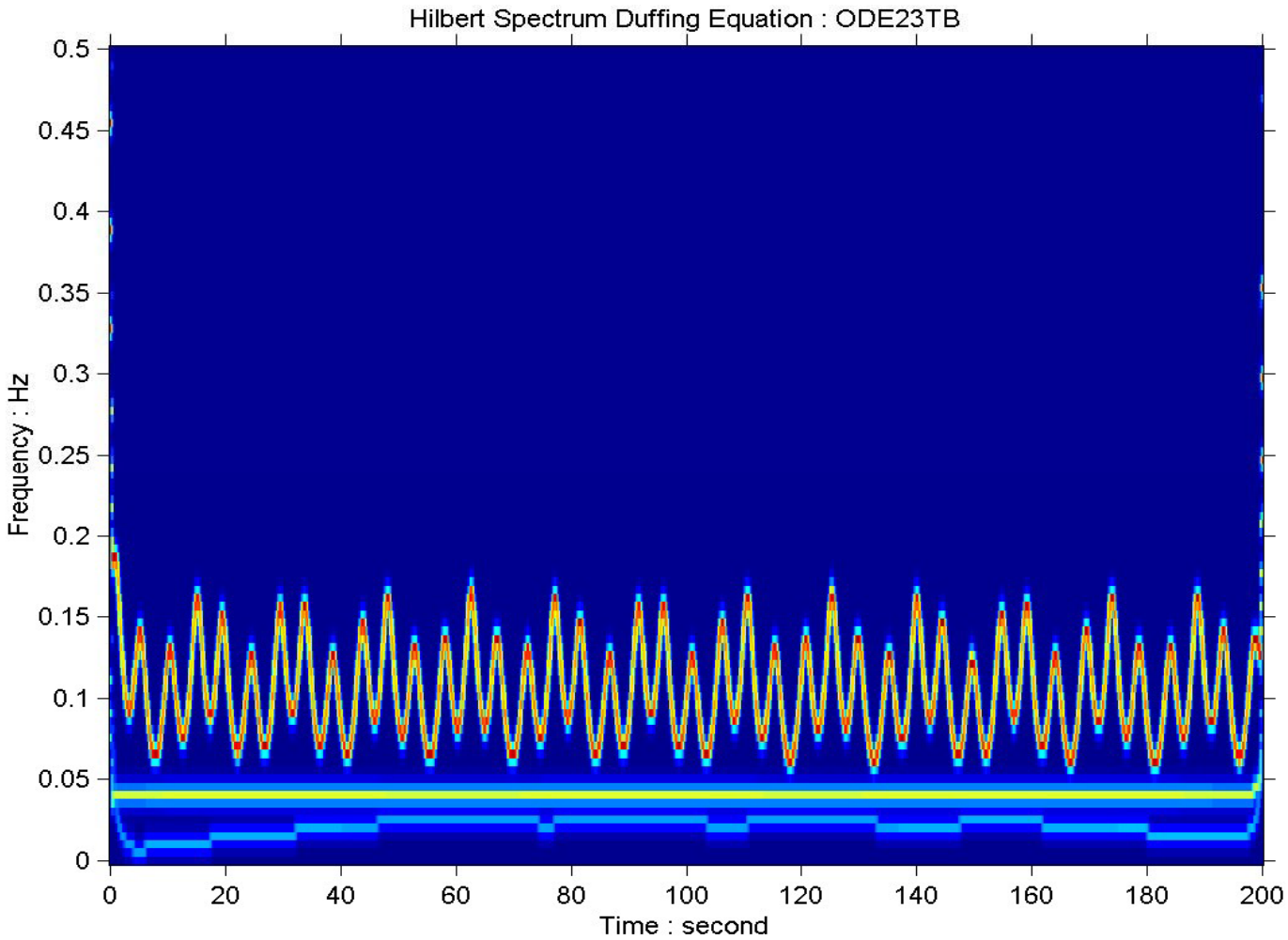
Duffing Equation : IMFs



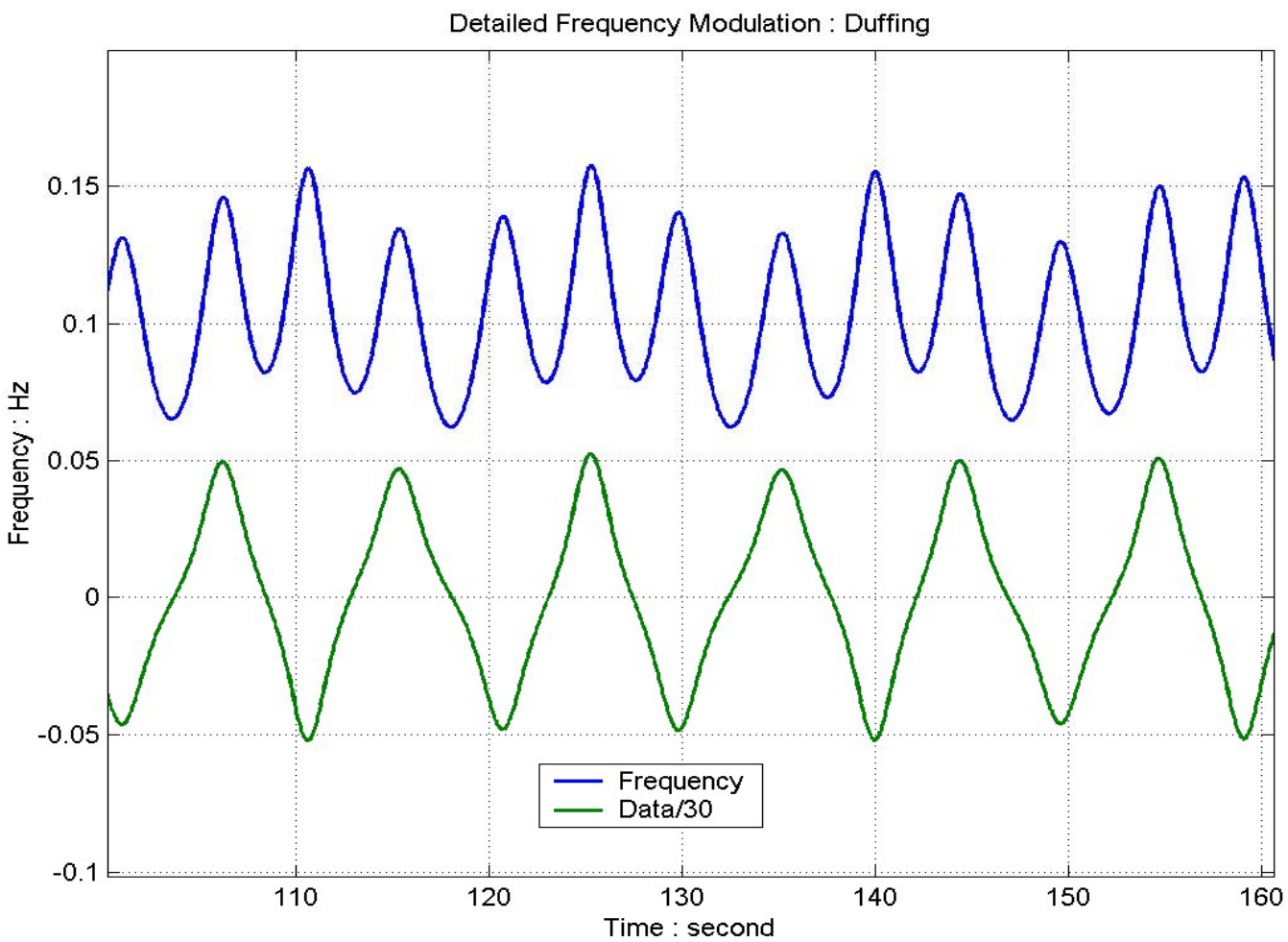
Duffing Equation : IMFs



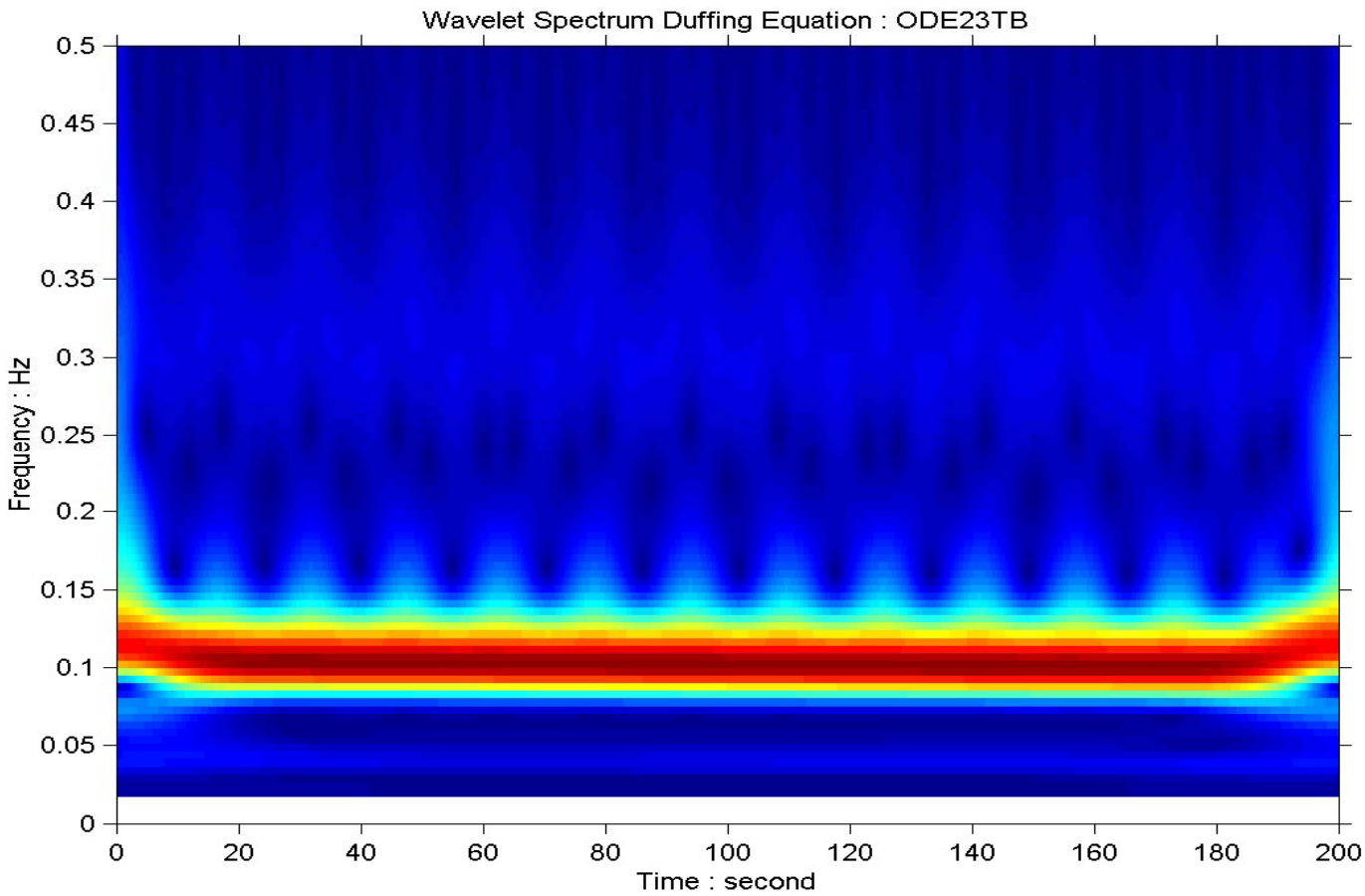
Duffing Equation : Hilbert Spectrum



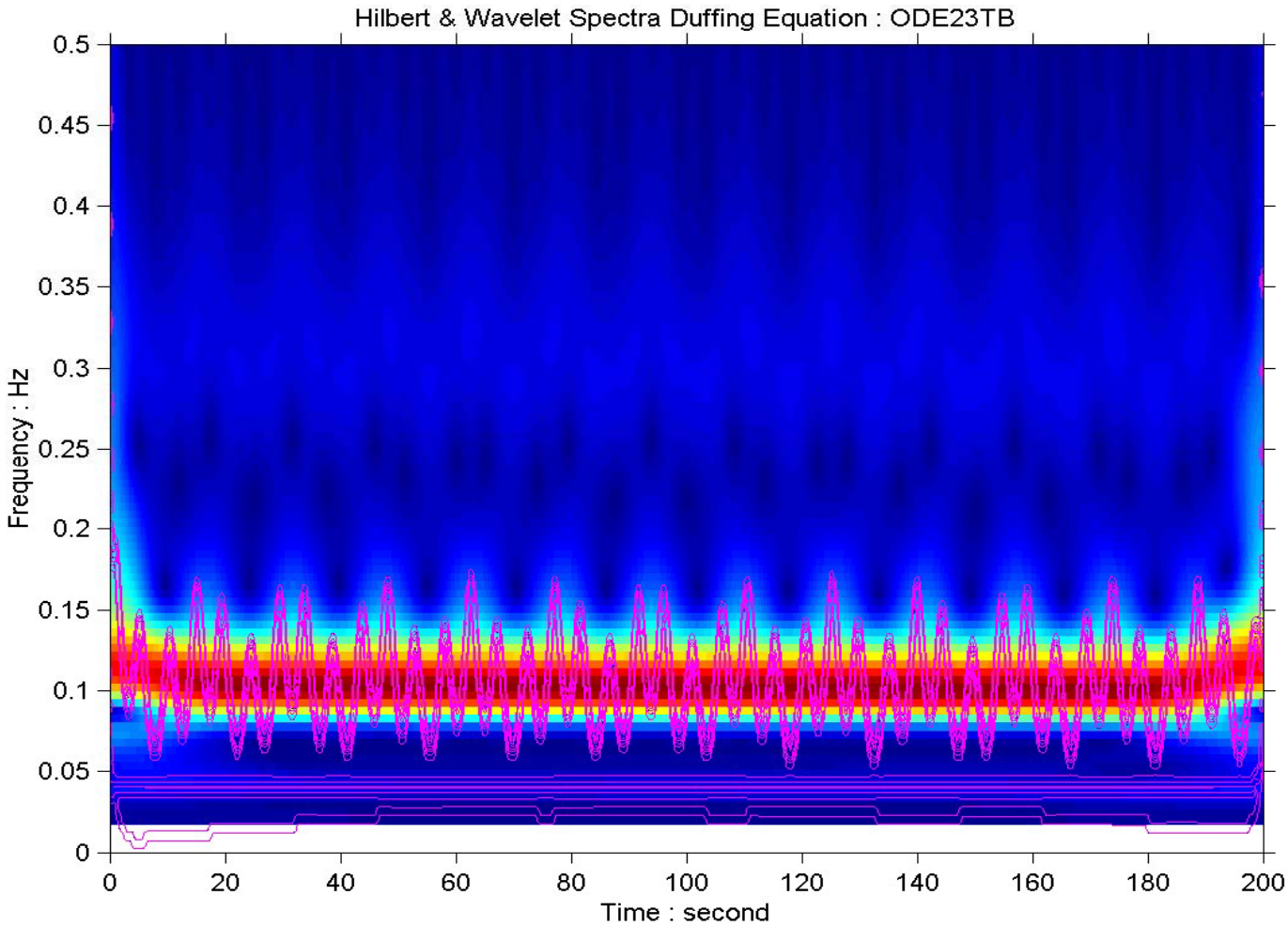
Duffing Equation : Detailed Hilbert Spectrum



Duffing Equation : Wavelet Spectrum



Duffing Equation : Hilbert & Wavelet Spectra



What This Means

- Instantaneous Frequency offers a total different view for nonlinear data: instantaneous frequency with no need for harmonics and unlimited by uncertainty.
- Adaptive basis is indispensable for nonstationary and nonlinear data analysis
- HHT establishes a new paradigm of data analysis

The Idea and the need of Instantaneous Frequency

According to the classic wave theory, the wave conservation law is based on a gradually changing ***φ(x,t)*** such that

$$\vec{k} = \nabla \varphi \ , \qquad \omega = -\frac{\partial \varphi}{\partial t} \ ;$$

$$\Rightarrow \quad \frac{\partial \vec{k}}{\partial t} + \nabla \omega = 0 \ .$$

Therefore, both wave number and frequency must have instantaneous values.

The Idea and the need of Instantaneous Frequency

According to the classic wave theory, there are other more important wave conservation laws for Energy and Action:

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left(C_g E\right) = 0 \; ; \; Energy$$

$$\frac{\partial}{\partial t}\left(\frac{E}{\omega}\right) + \frac{\partial}{\partial x_j}\left(\frac{C_{gj} E}{\omega}\right) = 0 \; ; \; Action$$

Therefore, if frequency is a function of time, it has to satisfy certain condition for both laws to be valid.

Prevailing Views

The term, Instantaneous Frequency, should be banished forever from the dictionary of the communication engineer.

J. Shekel, 1953

The uncertainty principle makes the concept of an Instantaneous Frequency impossible.

K. Gröchennig, 2001

Comparisons

	Fourier	Wavelet	Hilbert
Basis	a priori	a priori	Adaptive
Frequency	Convolution: Global	Convolution: Regional	Differentiation: Local
Presentation	Energy-frequency	Energy-time-frequency	Energy-time-frequency
Nonlinear	no	no	yes
Non-stationary	no	yes	yes
Feature extraction	no	discrete : no continuous: yes	yes
Harmonics	yes	yes	no

Current Applications

- Non-destructive Evaluation for Structural Health Monitoring
  - (DOT, NSWC, and DFRC/NASA, KSC/NASA Shuttle)
- Vibration, speech, and acoustic signal analyses
  - (FBI, MIT, and DARPA)
- Earthquake Engineering
  - (DOT)
- Bio-medical applications
  - (Harvard, UCSD, Johns Hopkins, and Southampton, UK)
- Global Primary Productivity Evolution map from LandSat data
  - (NASA Goddard, NOAA)
- Cosmological Gravitational Wave and Planets hunting
  - (NASA Goddard, and Nicholas Copernicus University, Poland)
- Financial market data analysis
  - (NASA and HKUST)

Example 1.
Structure Damping Studies

US Navy Submarine Structure:

Damping determined by one impact load for all the frequency responses

Calibration: Numerical Simulation of High Modal Density Transient System Response

$$x(t) = \mathrm{Re}[F(t)e^{i2\pi f_0 t}]$$

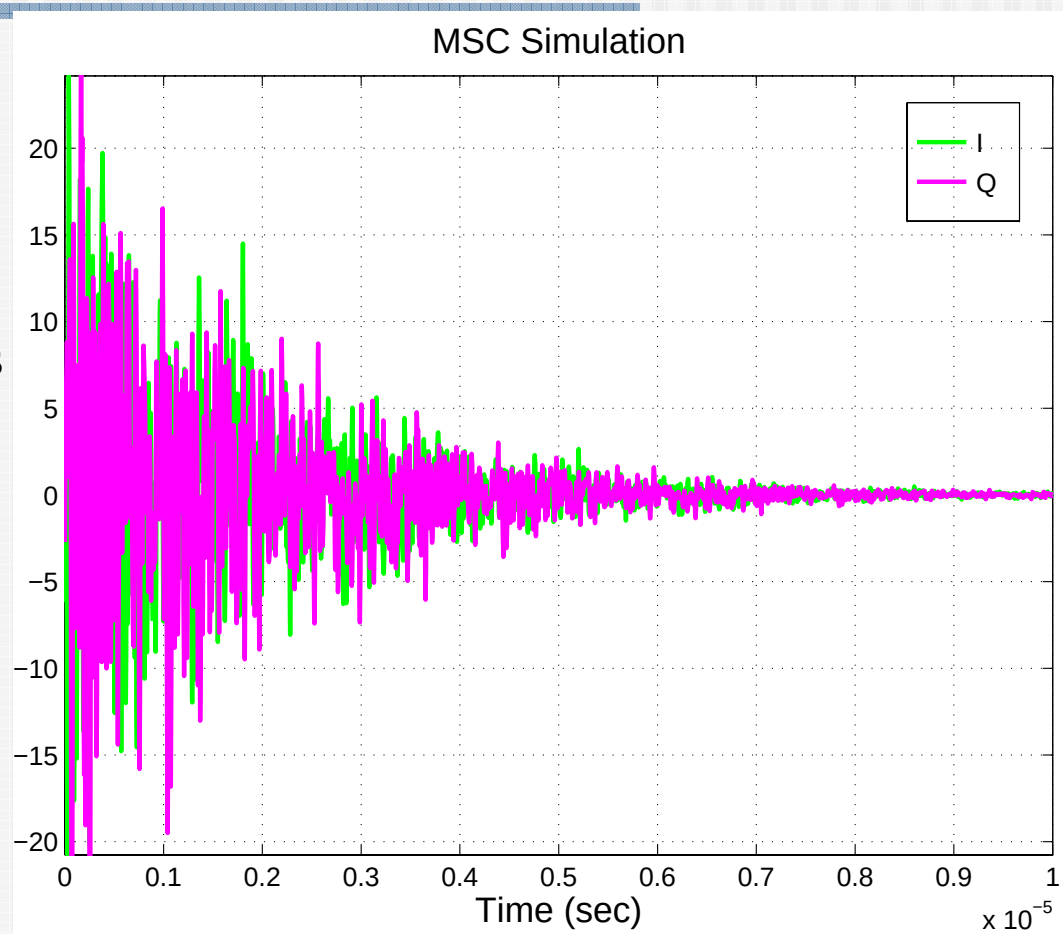
Where $$F(t) = \sum_{n=-\Delta n}^{n=\Delta n}{}' C_n e^{(i2\pi f_n - \gamma_n/2)t}$$

Signal is represented by a carrier frequency f_0, and its in-phase ***I(t)*** and quadrature ***Q(t)*** modulation components

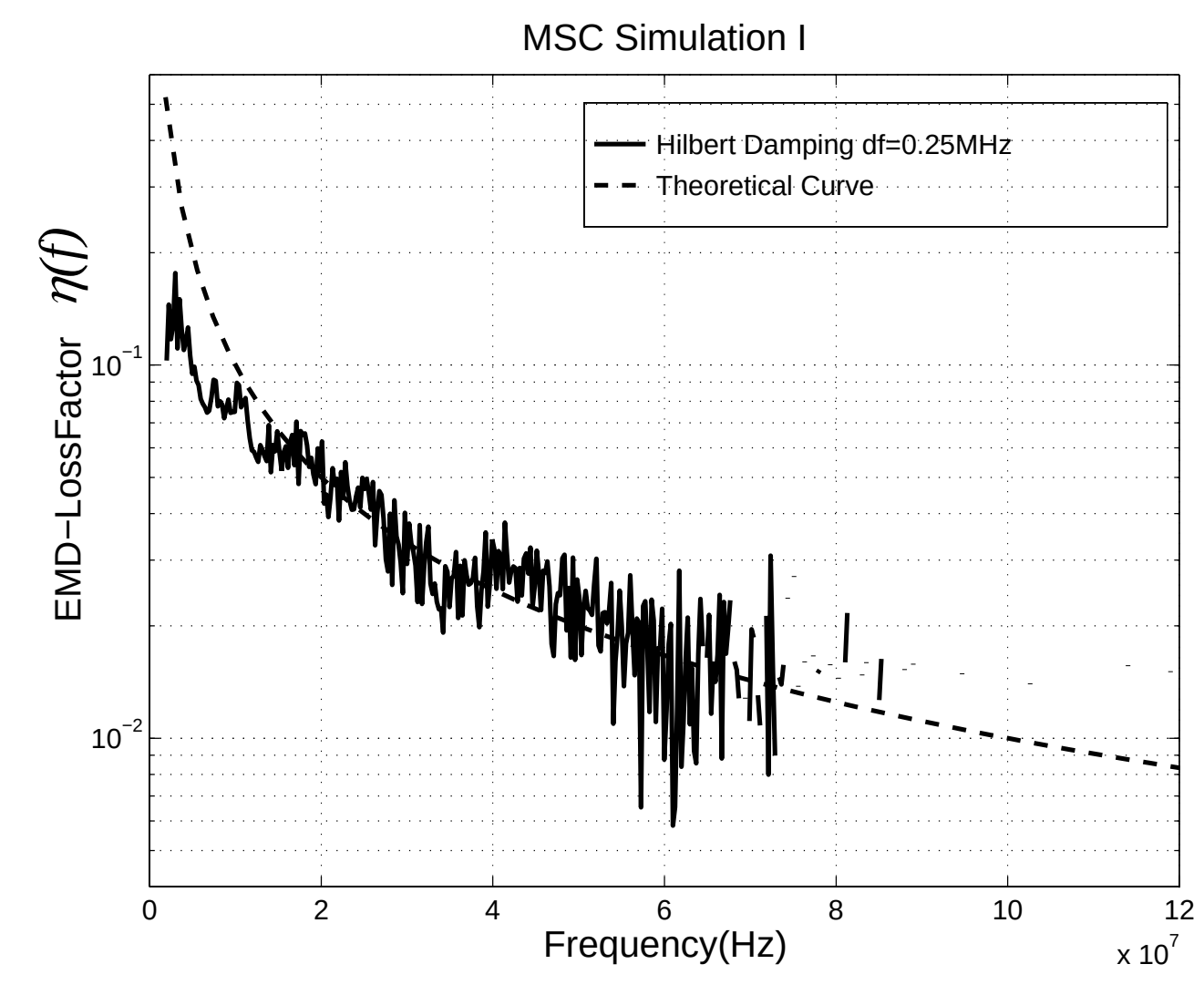
$$x(t) = I(t)\cos(2\pi f_0 t) - Q(t)\sin(2\pi f_0 t)$$

Example:

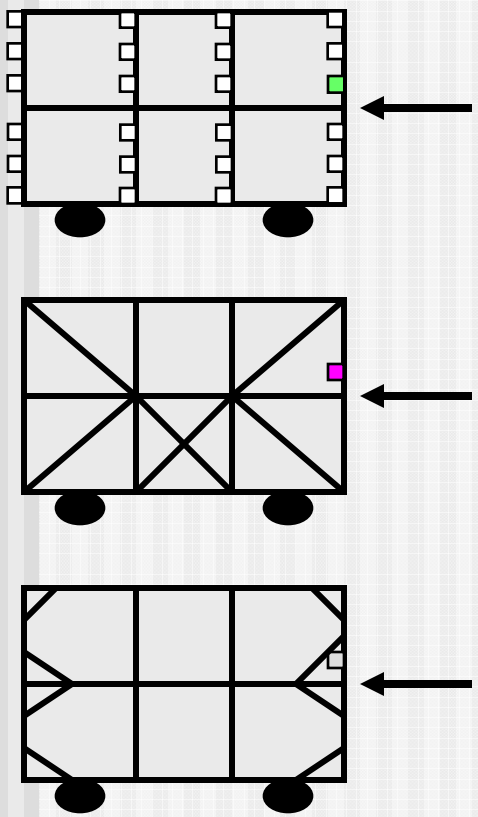
f_0 = 130 MHz, $\Delta f = 1/\Delta t \geq 50$ MHz
$\gamma = 10^6$ sec^{-1}
No. of spectrum lines: $2\Delta n = 340$
random phase
time series length: T = 10 msec
sampling Δt = 2 nsec

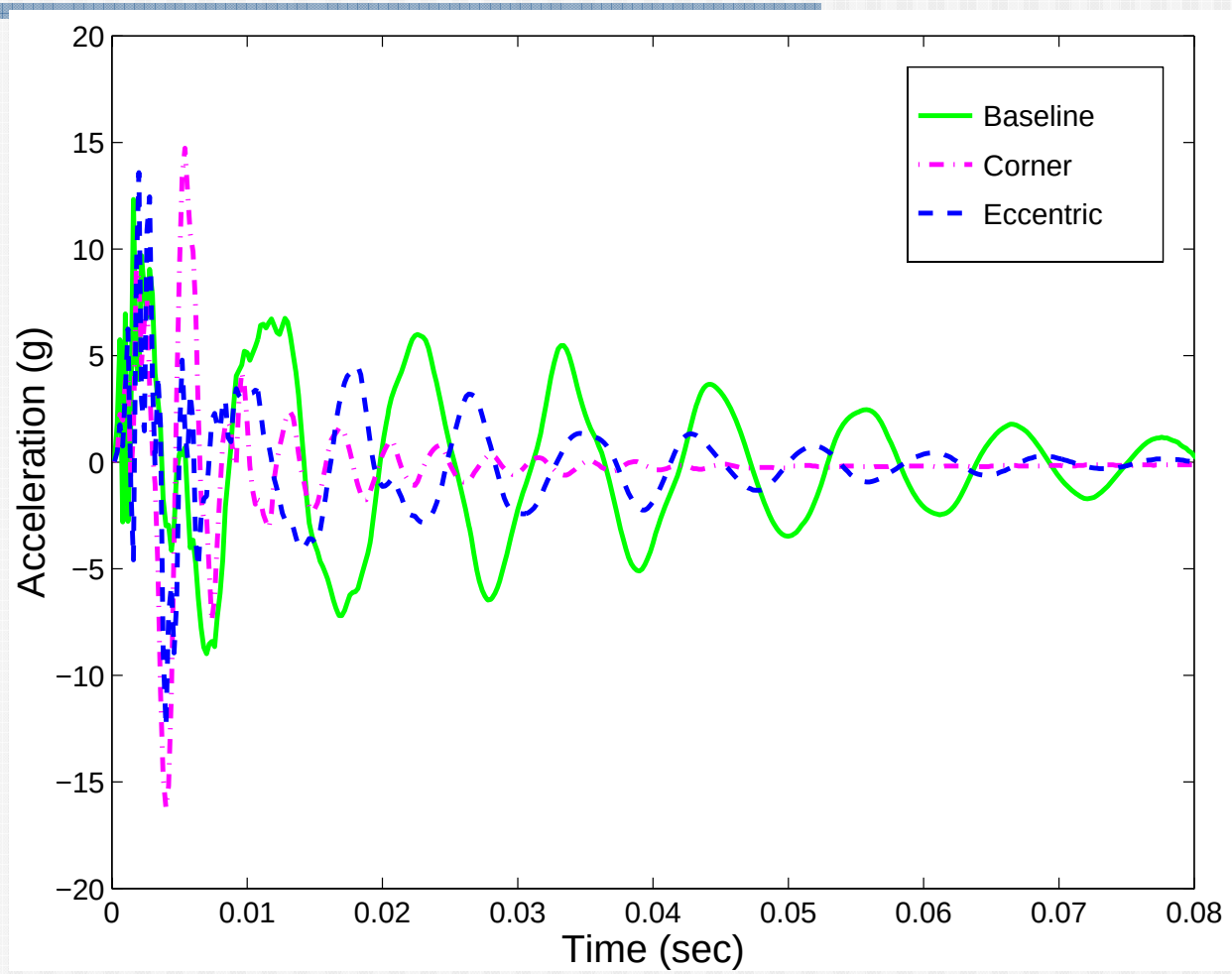


Determine Decay Rates for Simulation Data
Using Hilbert Damping Spectrum



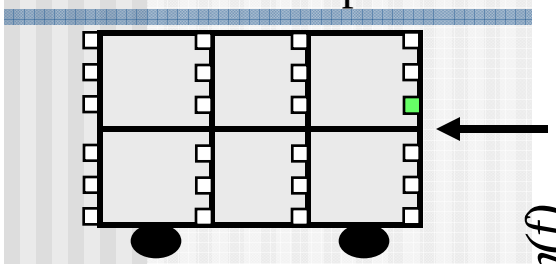
2D Frame Impact Point Time Series Baseline Frame vs Two Types of Diagonals



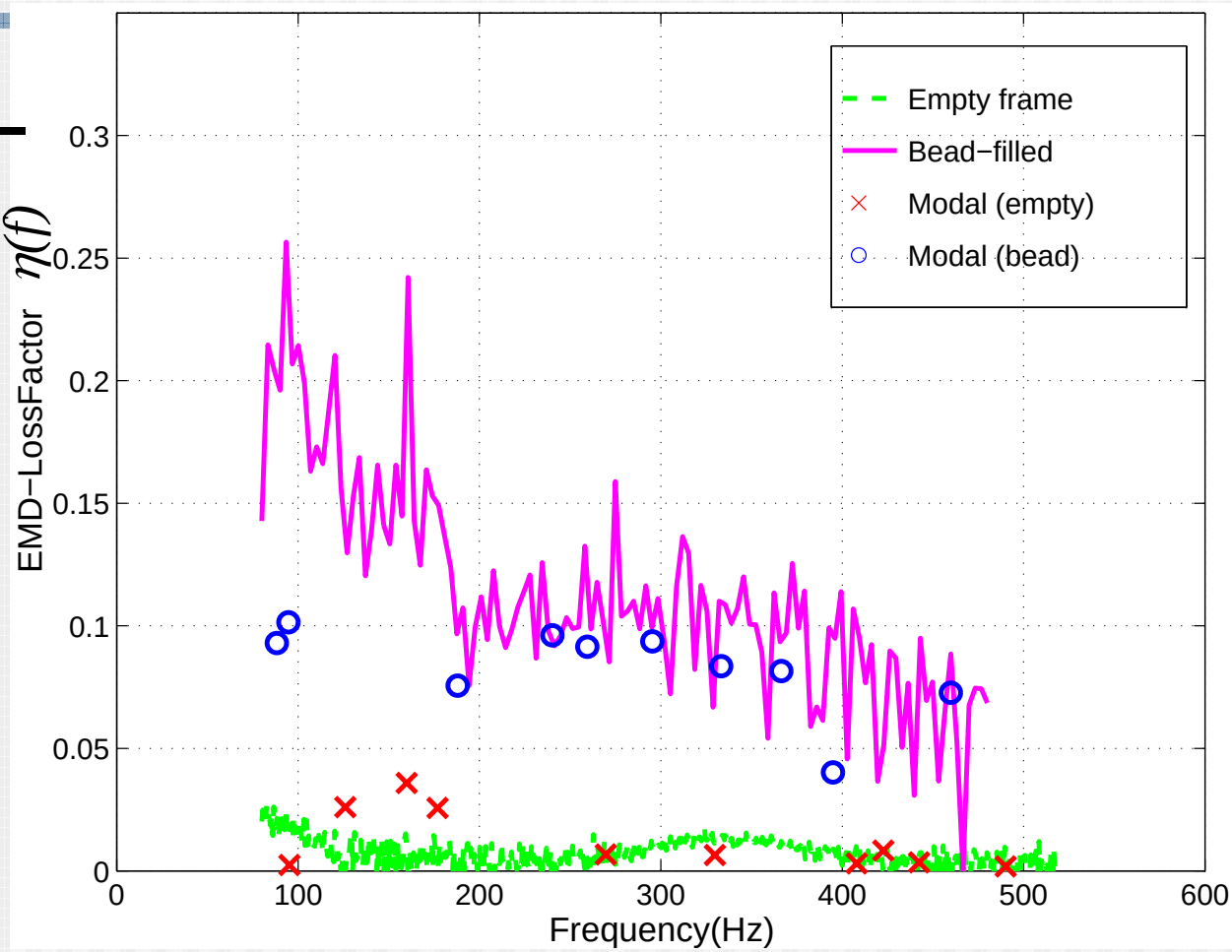


Loss Factor for Empty Frame and Bead-Filled Frame

Horizontal Impact:

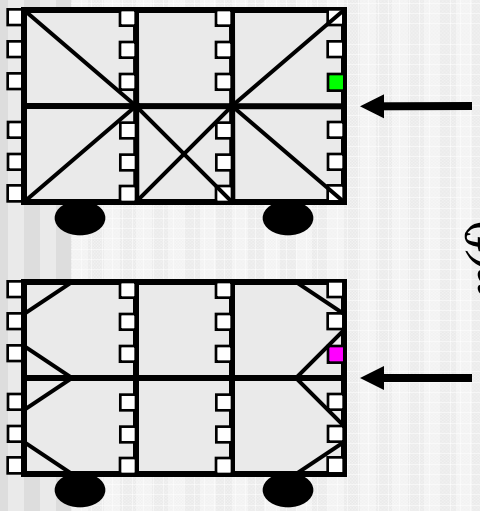


Comparison of loss factor calculated from modal method (Polyreference Time Domain algorithm) and empirical mode method

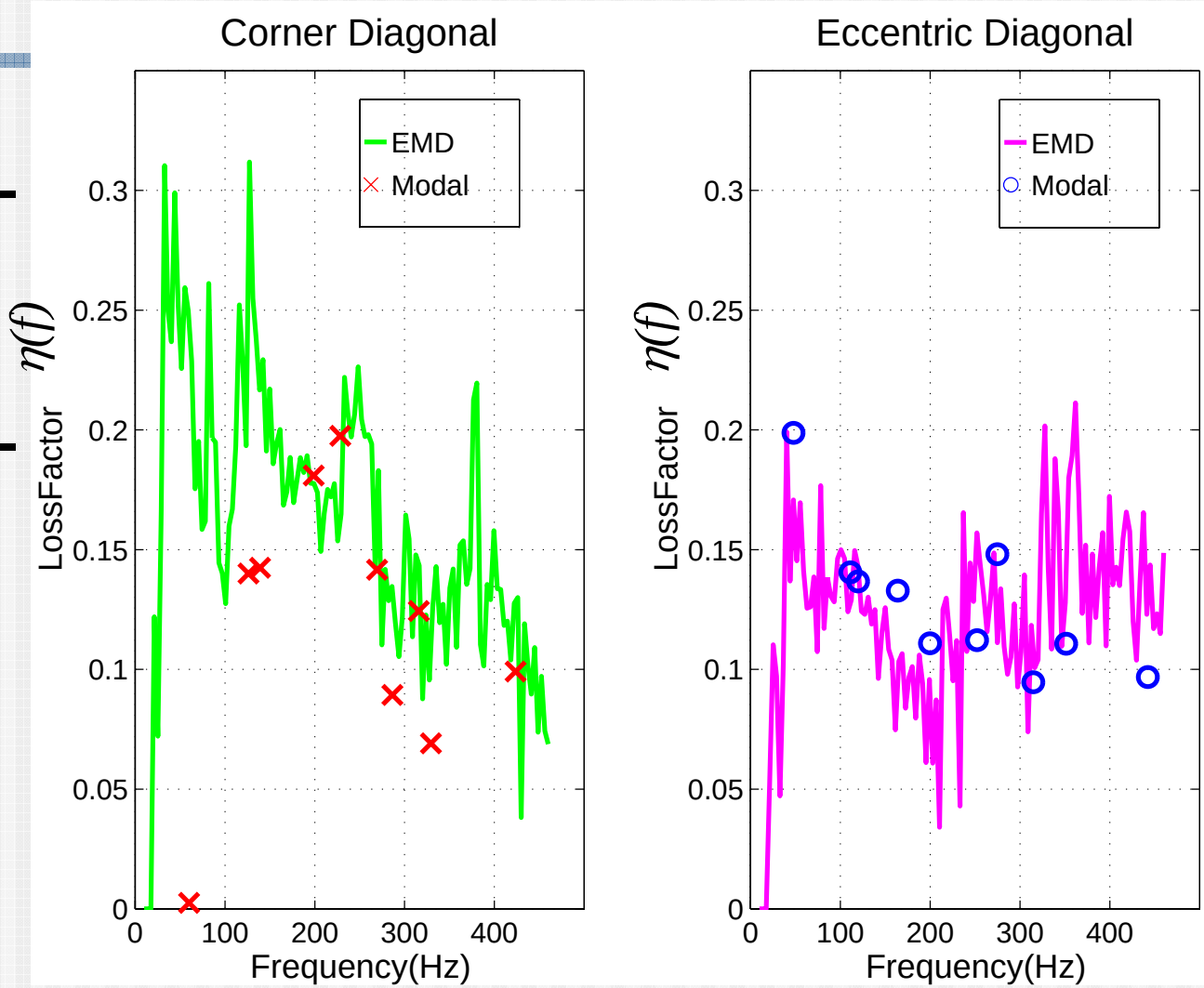


Loss Factor for Two Different Frame Construction Options

Horizontal Impact:



Comparison of loss factor calculated from modal method (Polyreference Time Domain algorithm) and empirical mode method



Example 2.
Speech Analysis

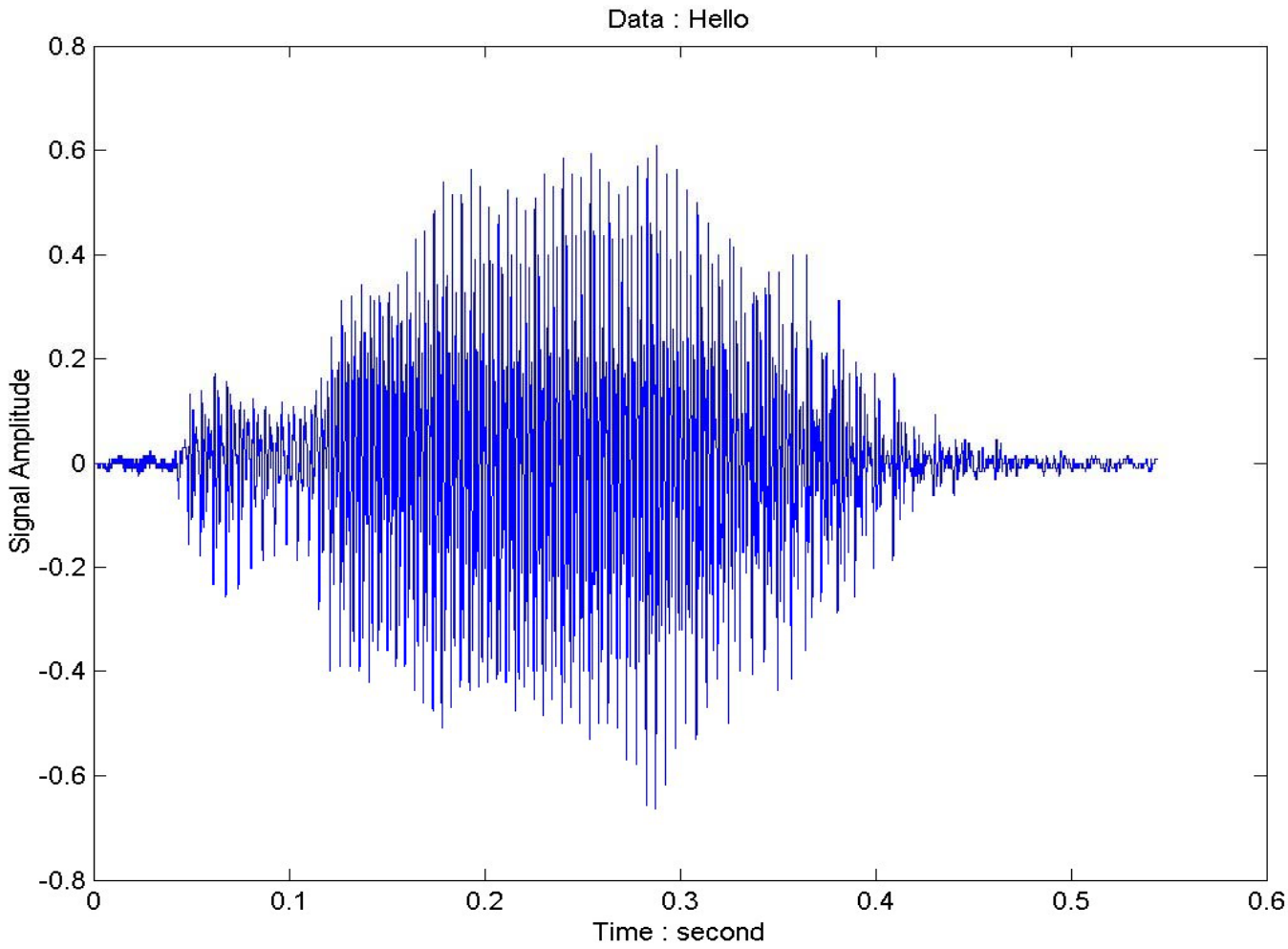
Comparisons among Different methods:

1. The Fourier Spectrograms Narrow band (Top left) and wideband (Top right)
2. The Wavelet Analysis (Bottom left)
3. The Hilbert Spectral Analysis (Bottom right)

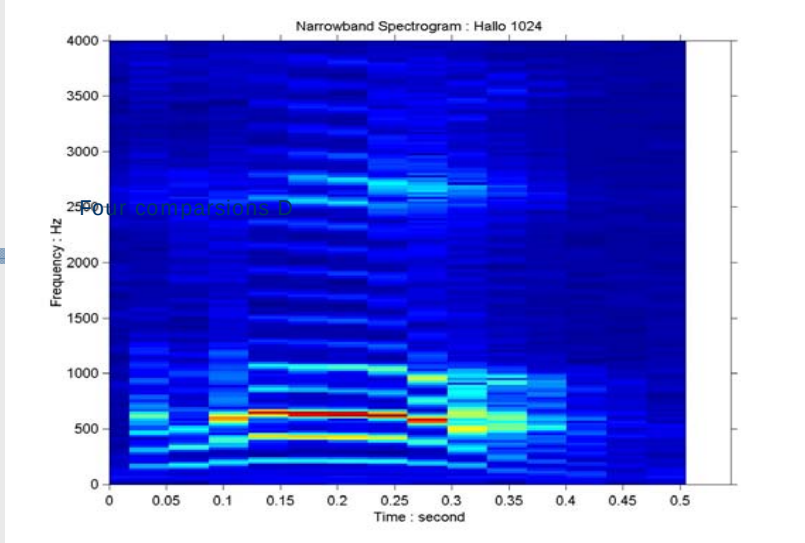
Notice the details of the Hilbert Spectrum

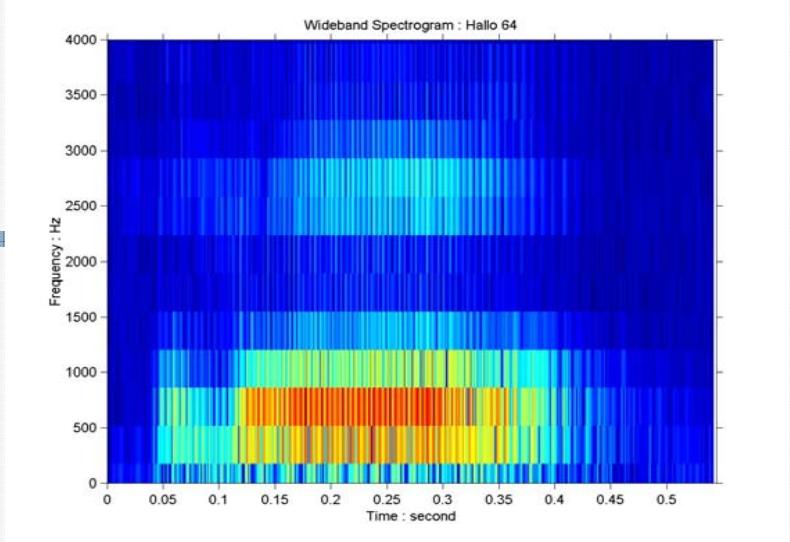
Speech Analysis

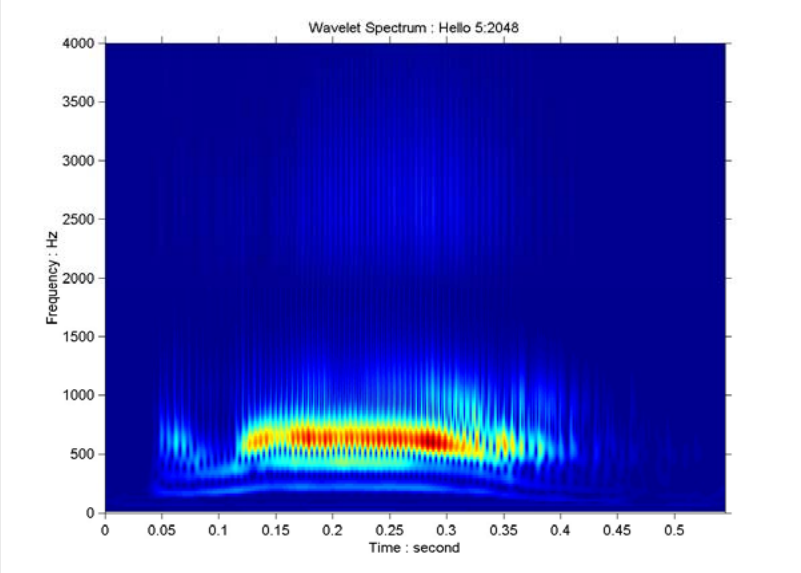
Hello : Data

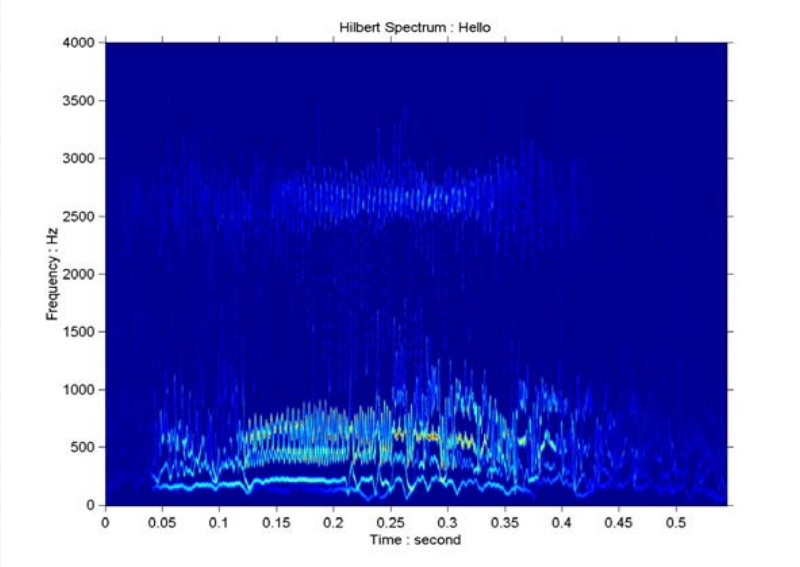


Four comparisons D

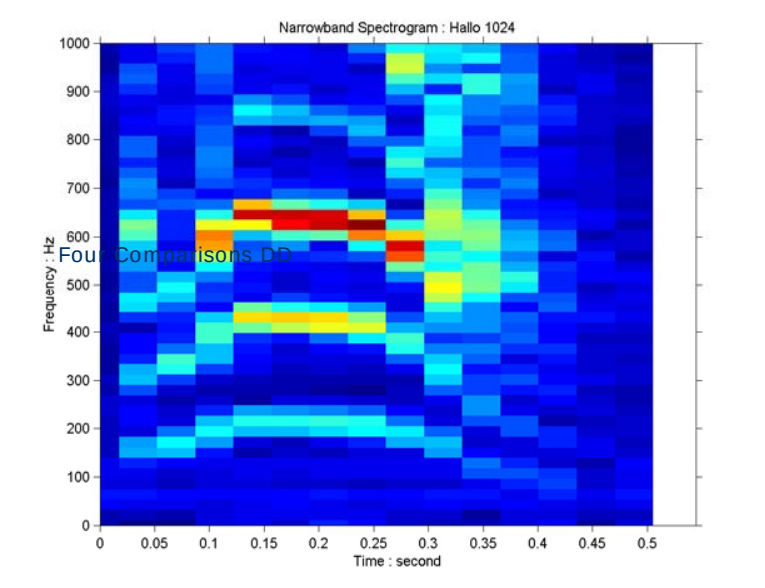


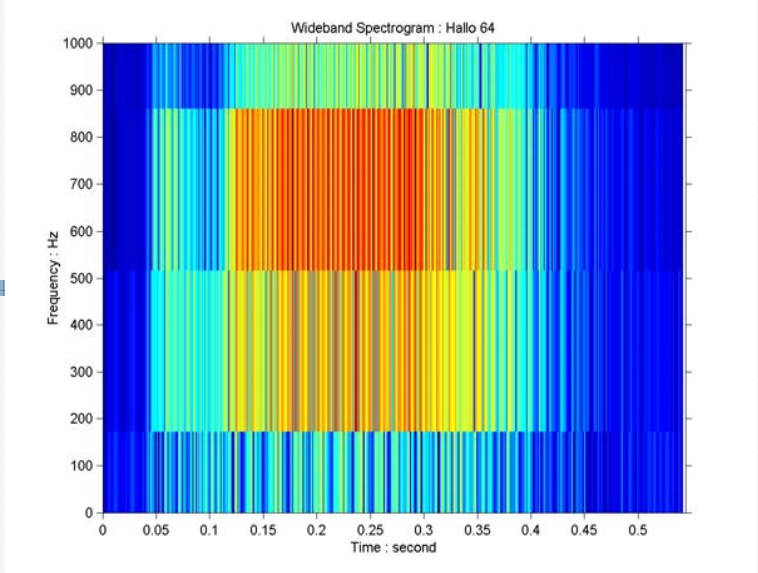


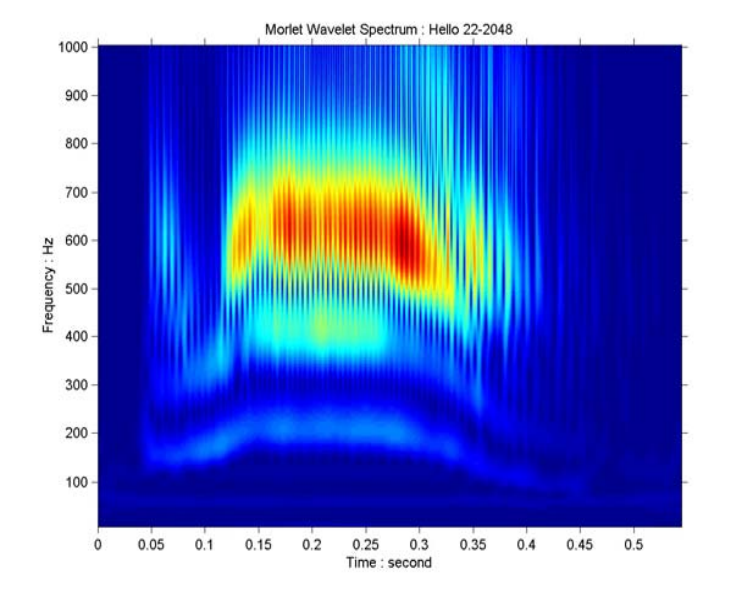


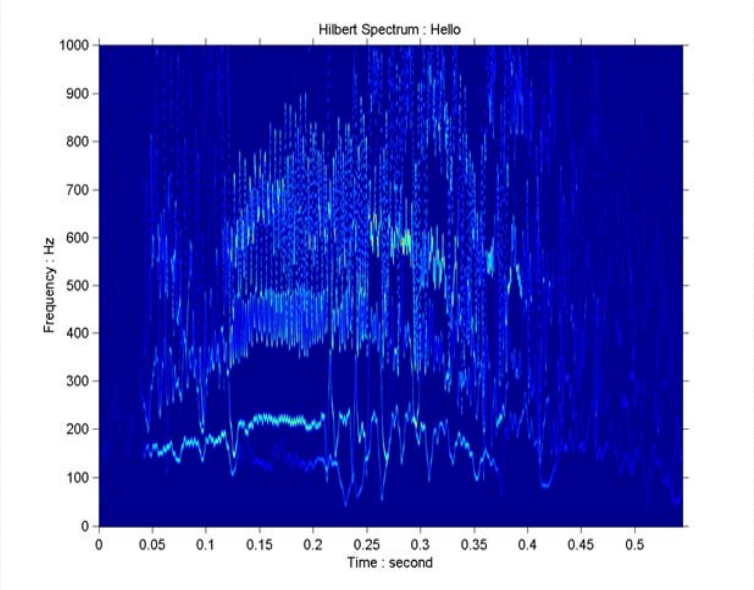


Four Comparisons DD







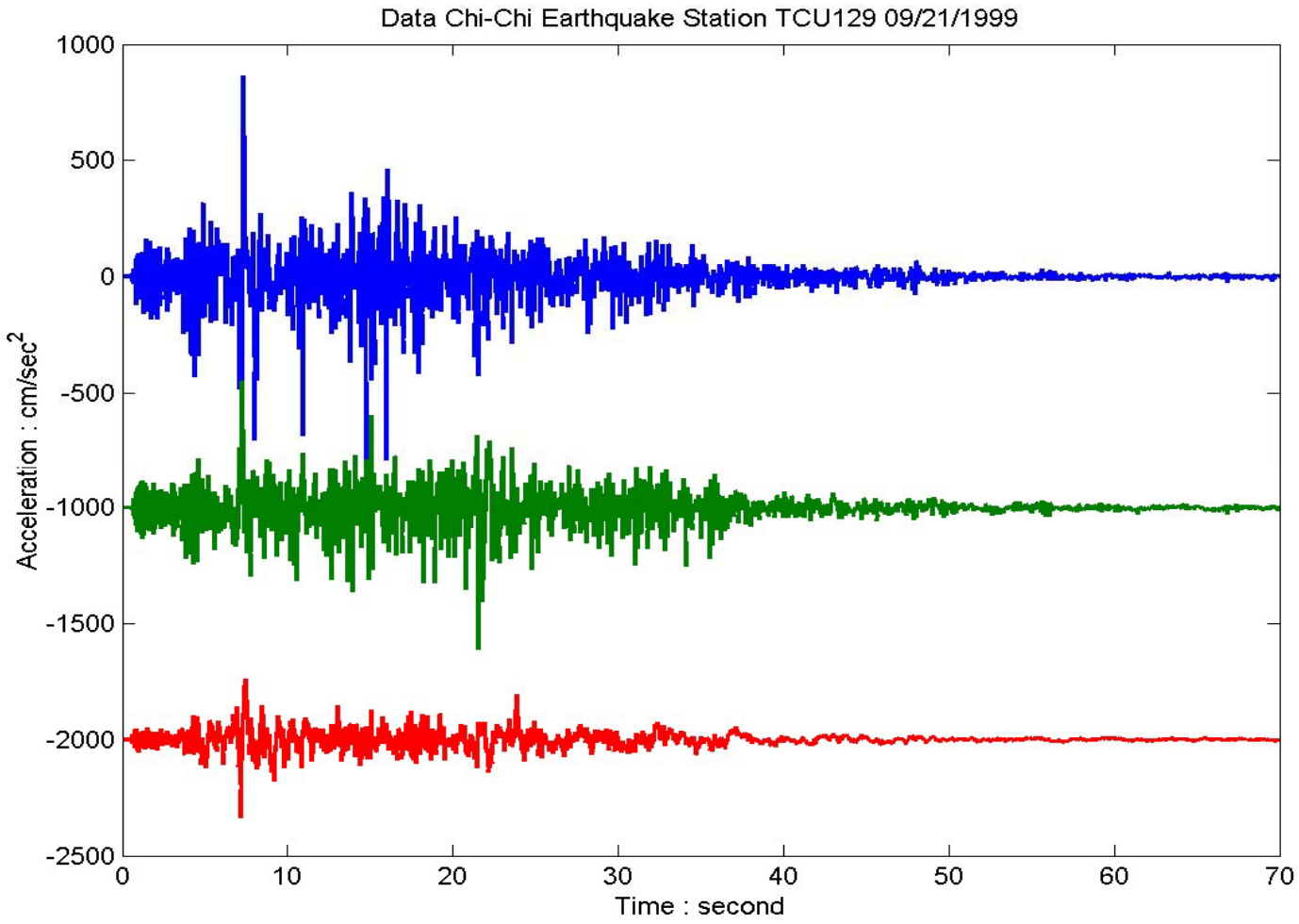


Example 3. Earthquake Analysis

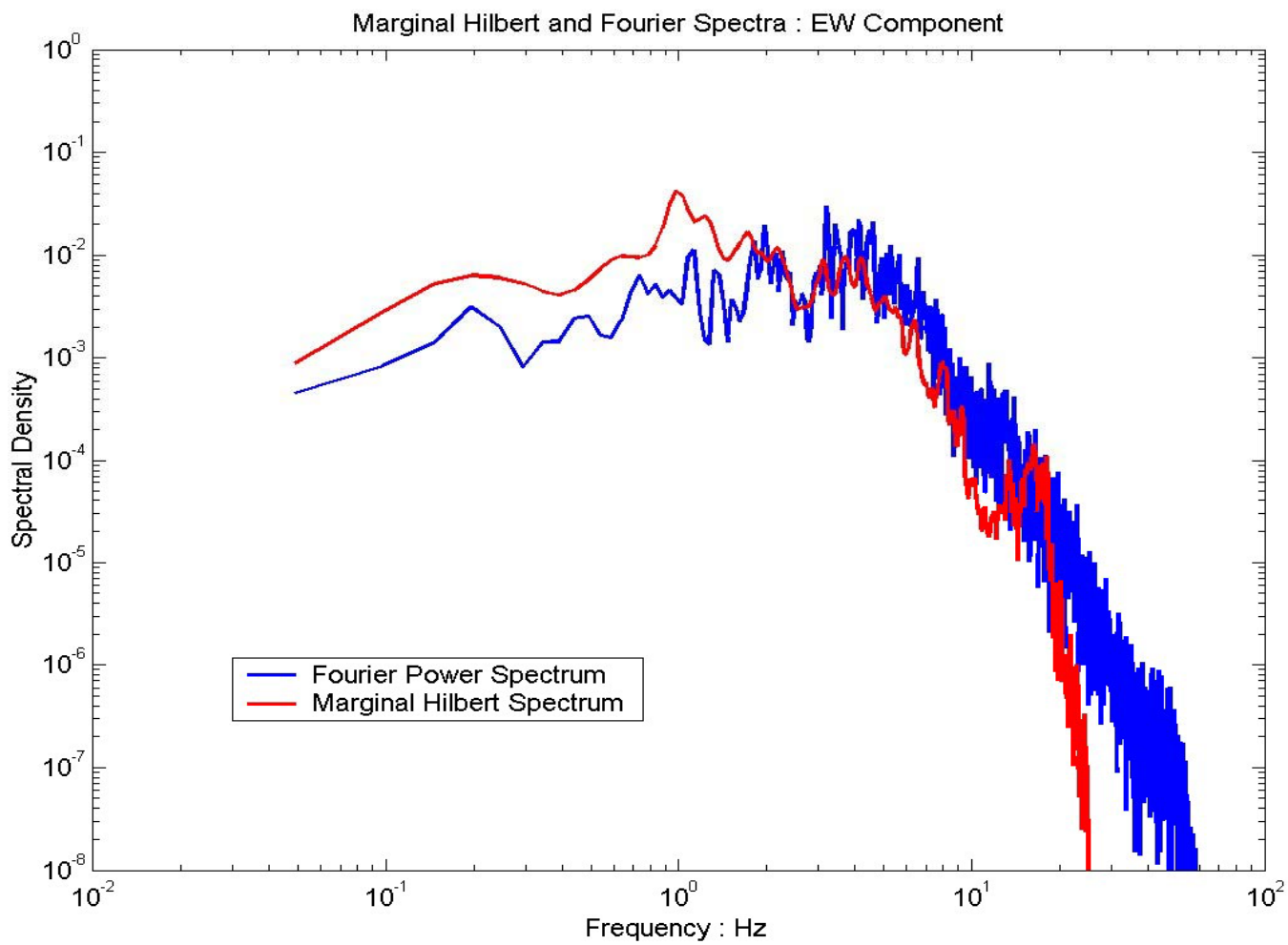
Chi-Chi earthquake data. Notice the difference between the Fourier and Hilbert Spectral Analysis:

1. Fourier under-represents energy of the low frequency range for lack of resolution.
2. Fourier over-represents of the high frequency range for the spurious harmonics.
3. Difference the largest in East-West component, which is the most nonlinear and nonstationary; smallest in vertical, which is the most stationary.

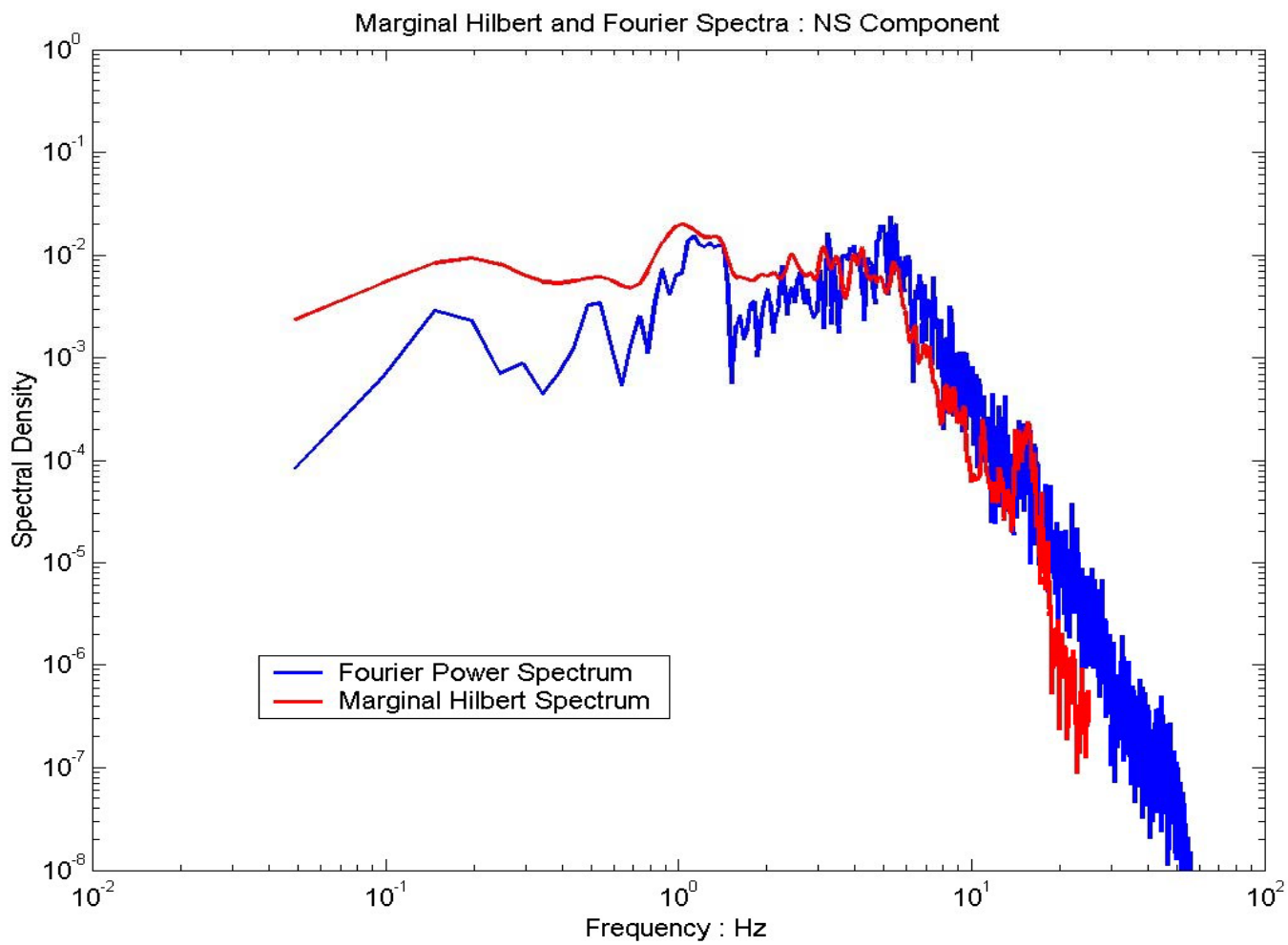
Chi-Chi Earthquake : Data



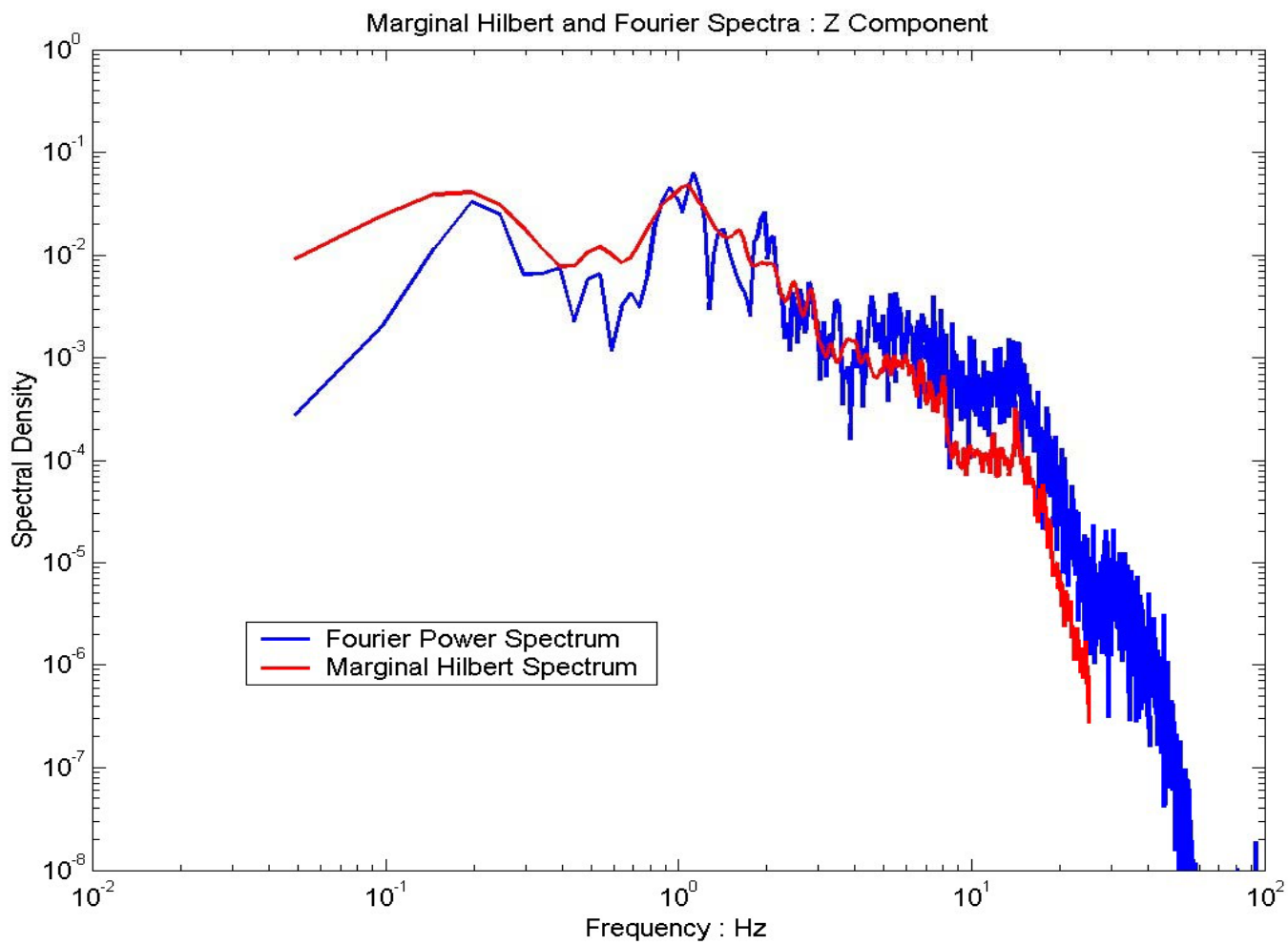
Chi-Chi Earthquake : East-West



Chi-Chi Earthquake : North-South



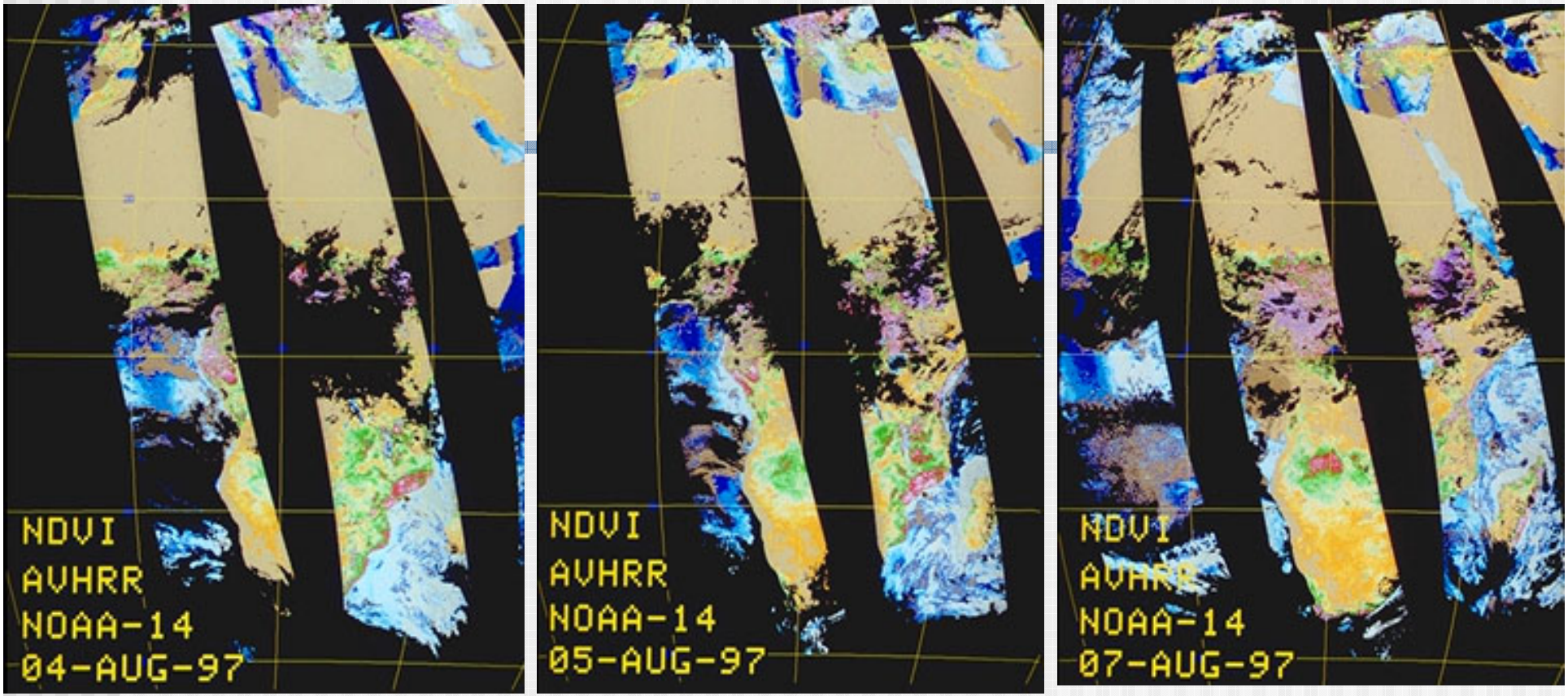
Chi-Chi Earthquake : Vertical



Example 4.
Removal of Satellite Orbit Drifts

For the first time, HHT correction of the orbit drifts enables us to construct a uniform long time sequence of the global primary productivity data from 6 different Satellite.

NDVI Maximum Value Composites

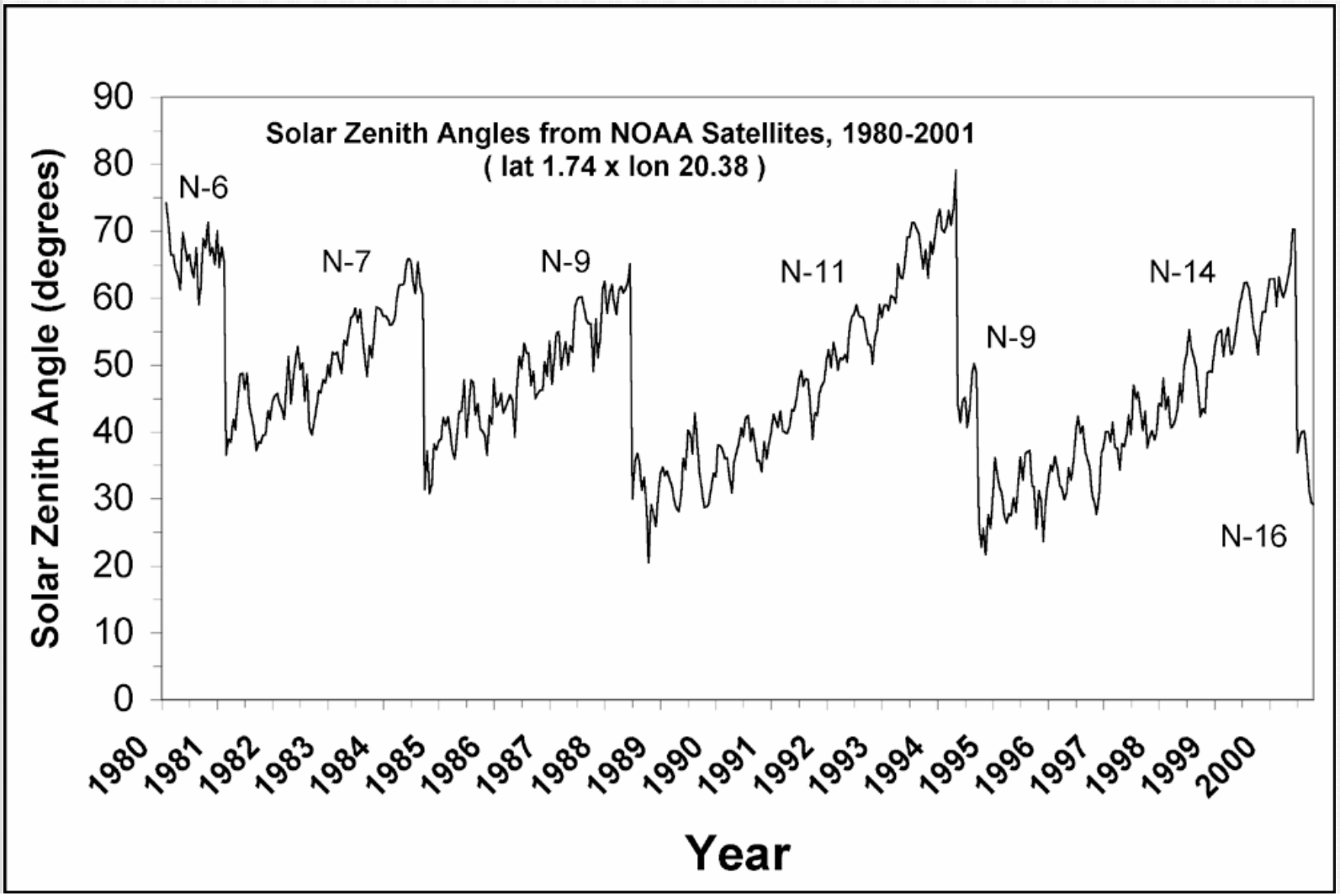


8-km output bin from 4-km spatial resolution data at subpoint

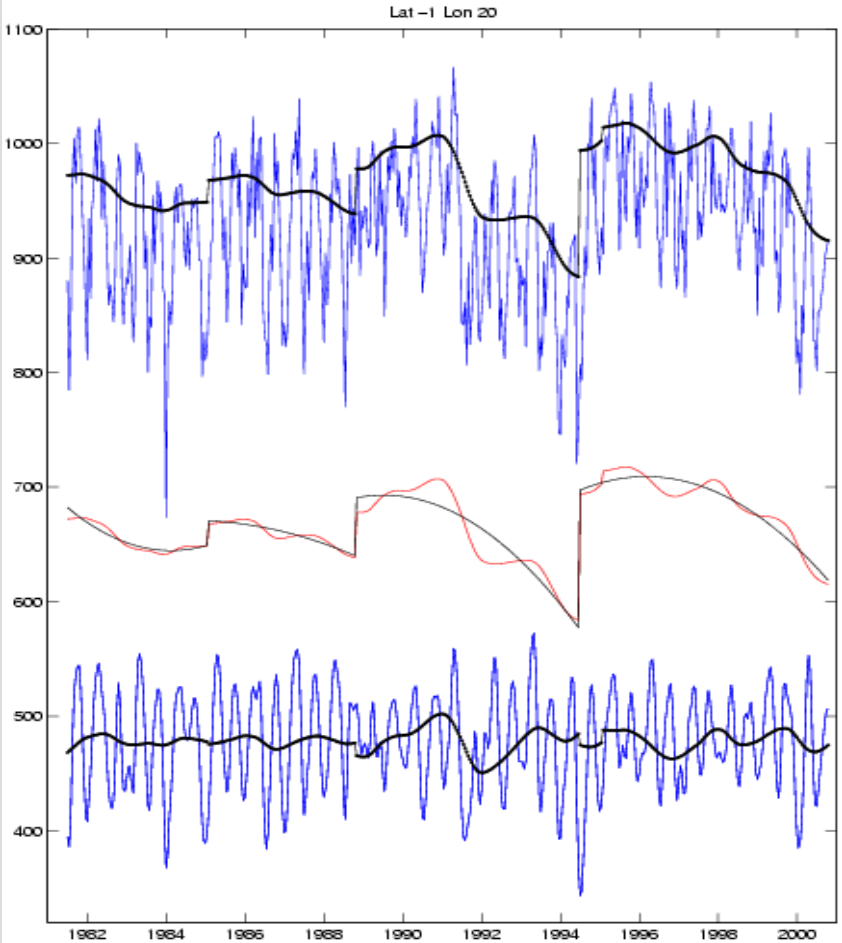
All orbits, all data processed, equal area map projection

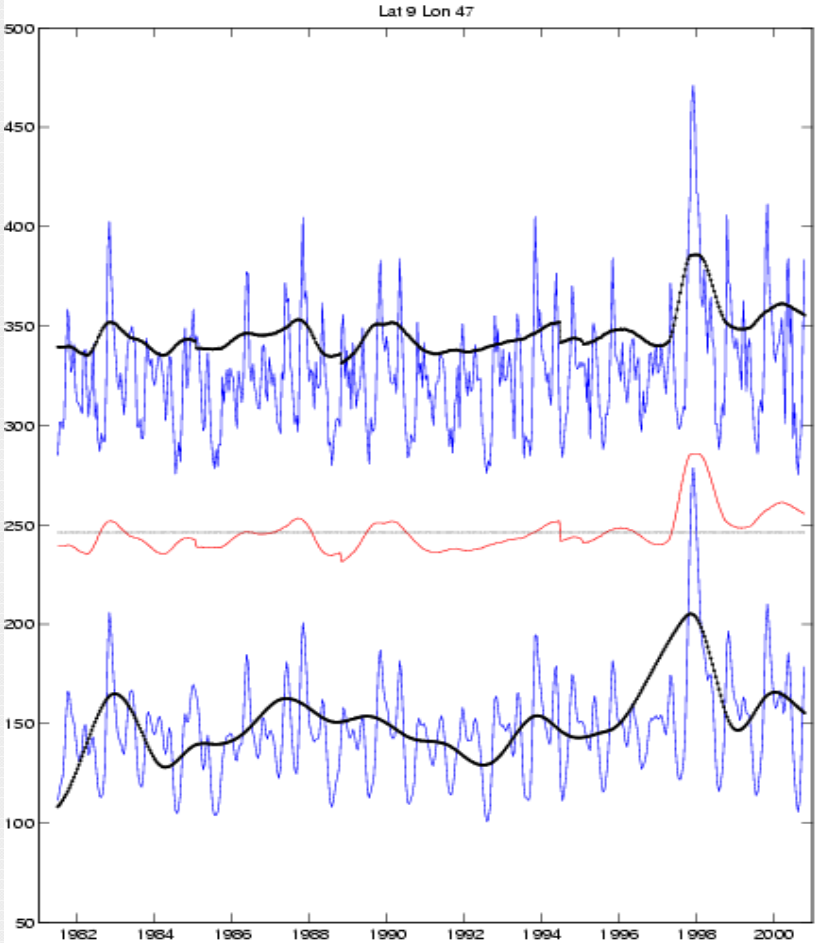
Scan Angle Restriction to +/- 40 degrees

AVHRR analyses require Solar zenith angle correction

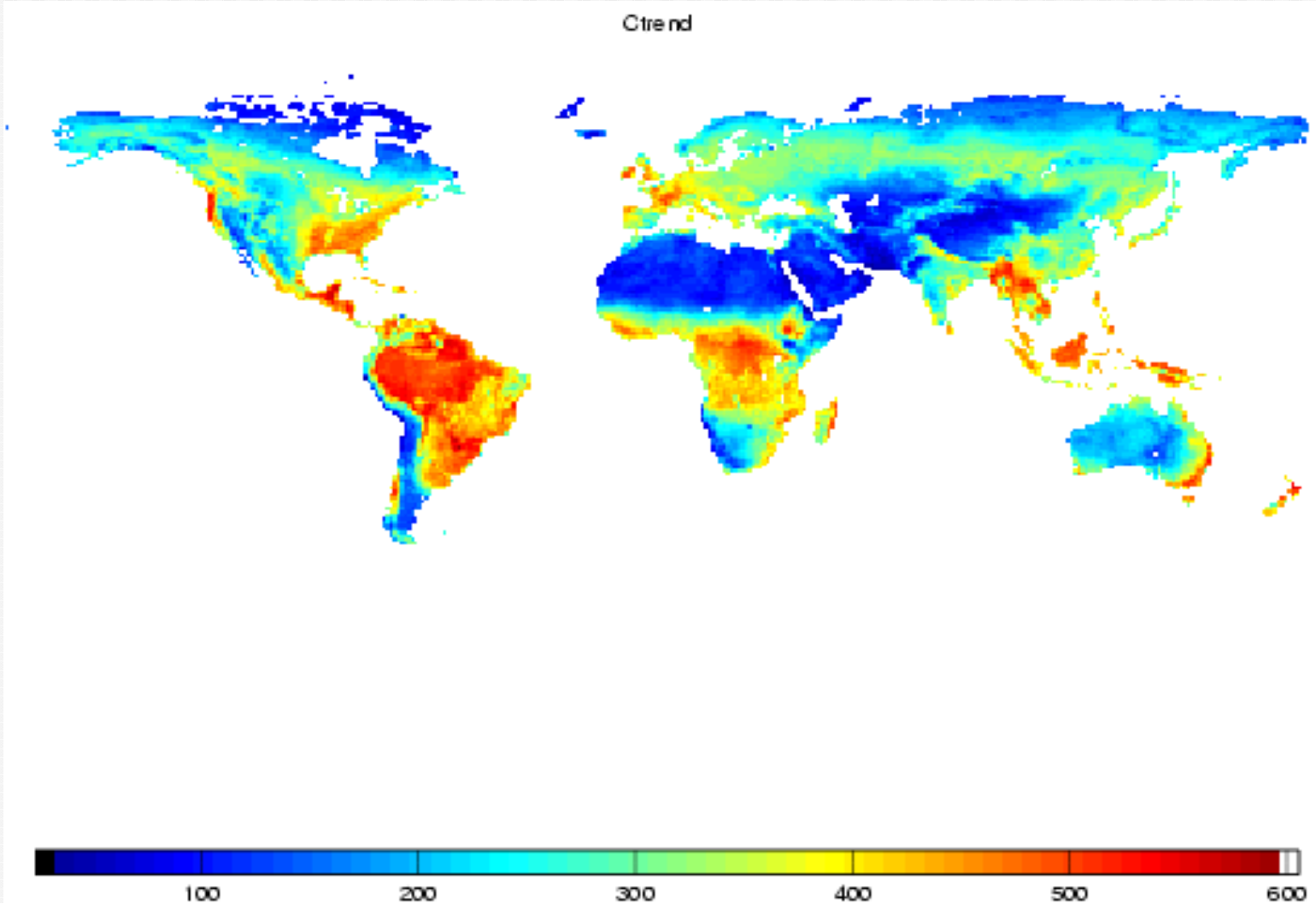


EMD processing

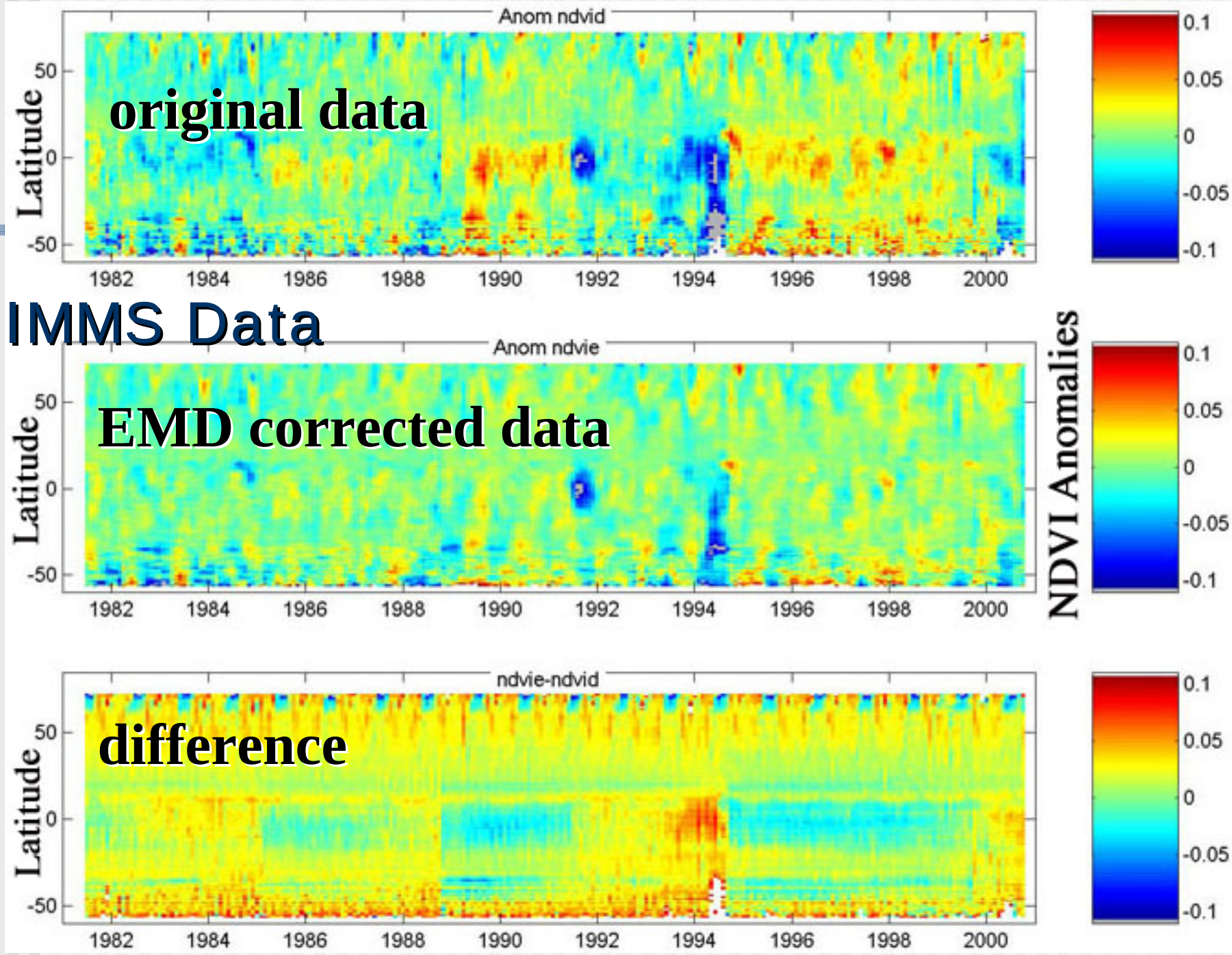




EMD Median Trend



GIMMS Data

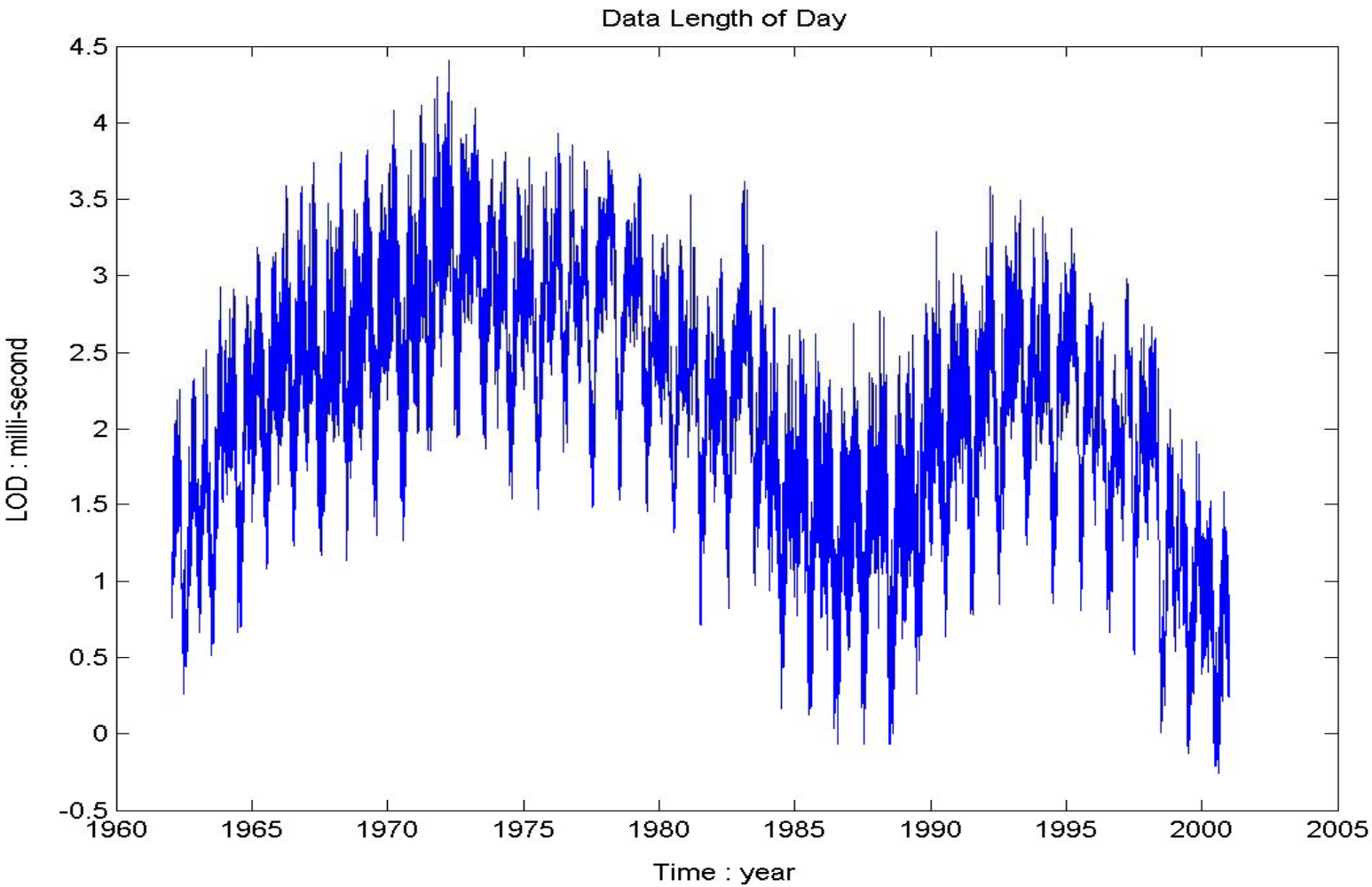

Anom ndvid
original data
Latitude
50
0
-50
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
0.1
0.05
0
-0.05
-0.1
Anom ndvie
EMD corrected data
NDVI Anomalies
ndvie-ndvid
difference

Example 5.
HHT Extract Climate Information

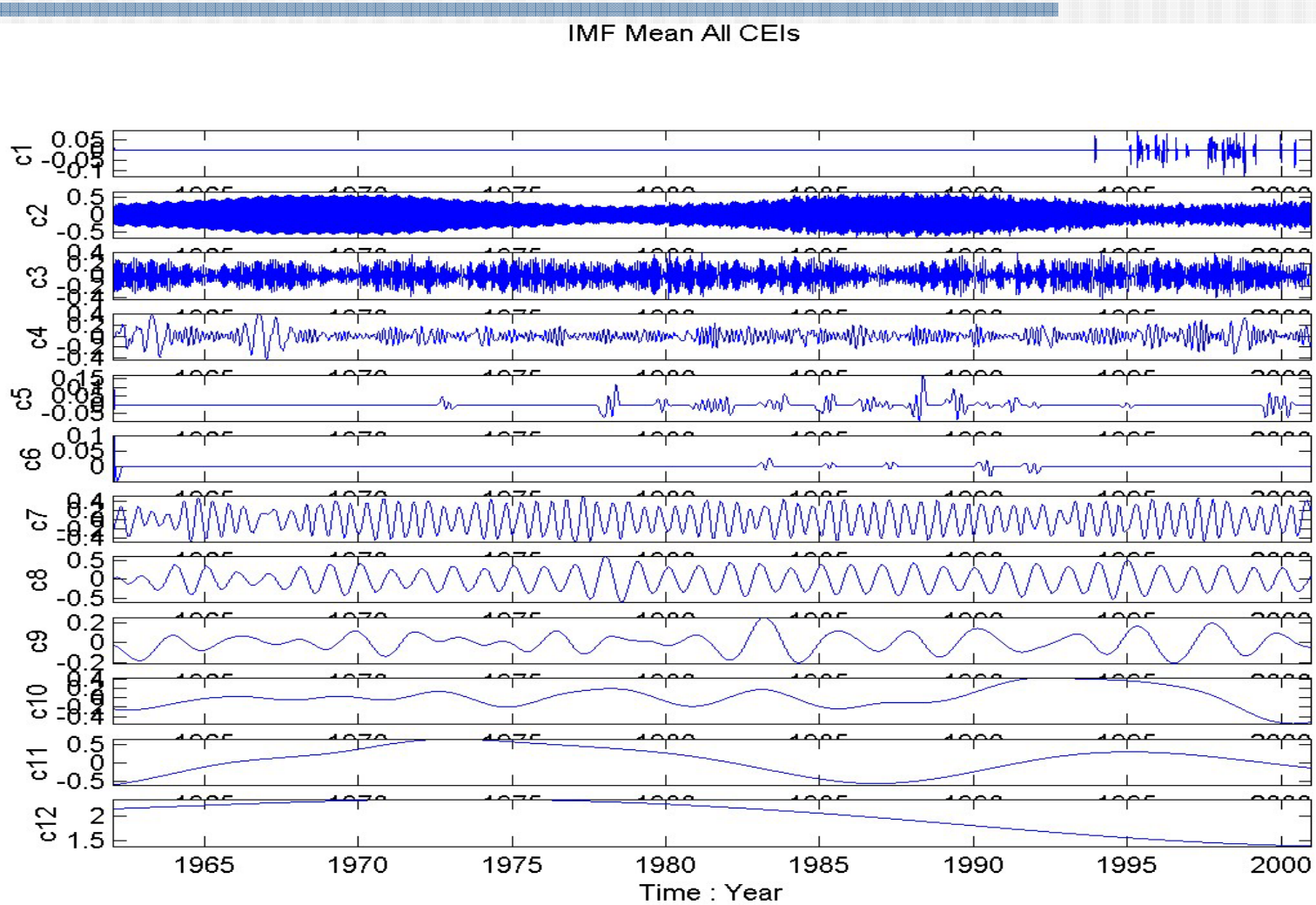
HHT enable us to extract the influence of El Nino on the annual cycle of the Earth rotating speed (as reflected in the length-of-day) data.

Each peak in the envelope represent an El Nino events.

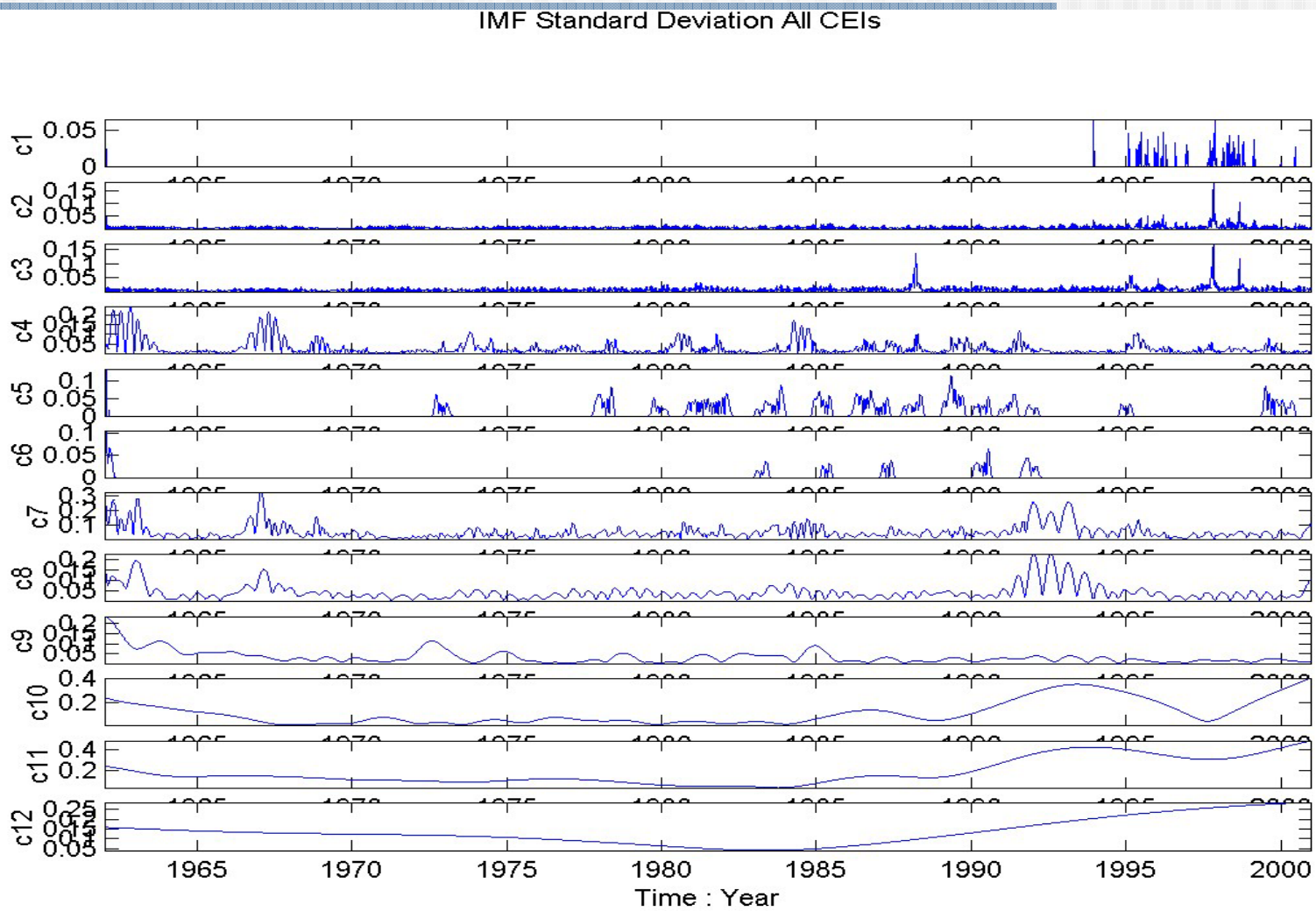
LOD Data


Data Length of Day
LOD : milli-second
Time : year
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005

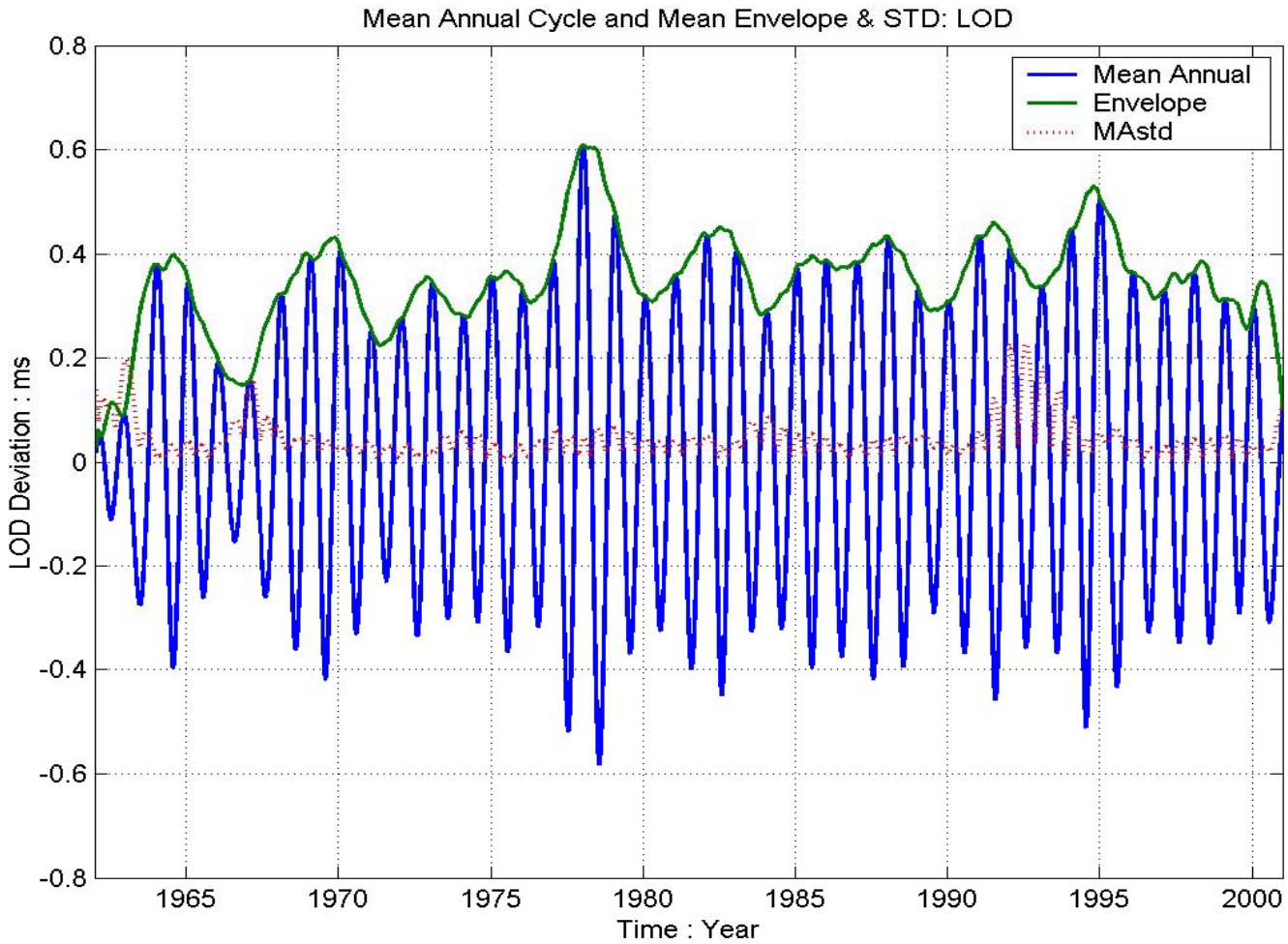
IMF : Mean CEI 9 cases



IMF : STD CEI 9 cases



Mean Annual Cycle & Envelope: 9 CEI Cases



Example 6.
Bridge Damage Detection

HHT detected the frequency shift of the pier vibration from 1.6 Hz to 1.2 Hz, which gives a ratio of 4:3. In terms of stiffness, the pier has suffered a ratio of 16:9 stiffness lost

The Fourier spectral analysis show no difference in Frequency shift.

A Bridge in Taiwan:

Notice the exposed piles

How do the bridges in US fare?

02/25/2001 17:13 6267959939 ENG. SCI. CALTECH PAGE 02

For Dr. Norden Huang 1-301-614-5644

NASA/GSFC 2/20/01

A5 (二)聞新國美 TUESDAY,

全美近三成橋樑有缺陷

公路管理署紀錄顯示 過於破舊有坍塌之虞 亟需修復

（美聯社華盛頓十九日電）聯邦一項探討全國道路系統現況的報告指出，全國四分之一以上的橋樑不夠堅實，過於破舊，或是無法承受目前的交通流量。

美聯社根據聯邦公路管理署的紀錄所做的分析顯示，截至二〇〇〇年八月卅一日，全國五十八萬七千七百五十五座橋樑當中，有十六萬七千九百九十三座（百分之廿九）被政府評為「有缺陷」，意即需要修復，橋面過窄或不足以承受往來交通。而夏威夷州、蒙德拿州及麻州，超過一半的橋樑被評為「有缺陷」。

聯邦政府一九九一年實施的跨國交通法案在七年內撥款一百六十一億元，後一年的經費為兩百零四億元。該報告指出，雖然過去幾年政府投資數十億元整修各地橋樑，但混凝土剝落、無法承受車輛負荷等狀況仍然層出不窮。阿拉巴馬州西南部蒙哥馬利有一座年久失修、過去校車上橋前，必須先讓學生下車走到橋的另一端上車。如今校車為繞開這座橋，得多走十五哩路。路易斯安納州橫跨湖背森林十年期一百卅億元的交通計畫，為二〇〇一年度預作準備的重他州自一九九六年起在修復或更換橋樑的開支上百萬元。該州以一九九七年增收的汽油稅及州府一般經費成立了百年公路基金。

其他許多州也都增加相關經費，例如北卡羅萊納州有四分之一的橋樑待修，州議會在一九九九年通過一項為期十年、總額達十五億元的橋樑修復計畫，北卡每年花四千五百萬元進行一般橋樑維護工程。該州三分之一的橋樑有缺陷。

北卡羅萊納州交通廳的負責人表示，他們要求六千五百萬元修護工程落後的橋樑。北卡每年花四千五百萬元進行一般橋樑維護工程。該州三分之一的橋樑有缺陷。

蒙哥馬利市政府剛附近的七十號州際公路的陸橋也發生桌球大小的水泥塊剝落的情況。

郡分類評為「有缺陷」的橋樑有坍塌之虞。一九九九年十月，麻州橫跨沙賓柏塔河的手樓突然崩塌，幸然無人受傷。當局仍將此橋封鎖修復。密西根州在塌陷前，早已禁止卡車通行。

威斯康辛州密爾瓦基市密西根湖上的雷茨橋因為去年十二月北向通路三支橋墩中的兩支出現裂痕而關閉。兩線車道加強結構後供雙向車輛通行。橋於廿七日恢復通行。

北卡羅萊納州交通廳的負責人表示，他們要求六千五百萬元修護工程落後的橋樑。北卡每年花四千五百萬元進行一般橋樑維護工程。該州三分之一的橋樑有缺陷。

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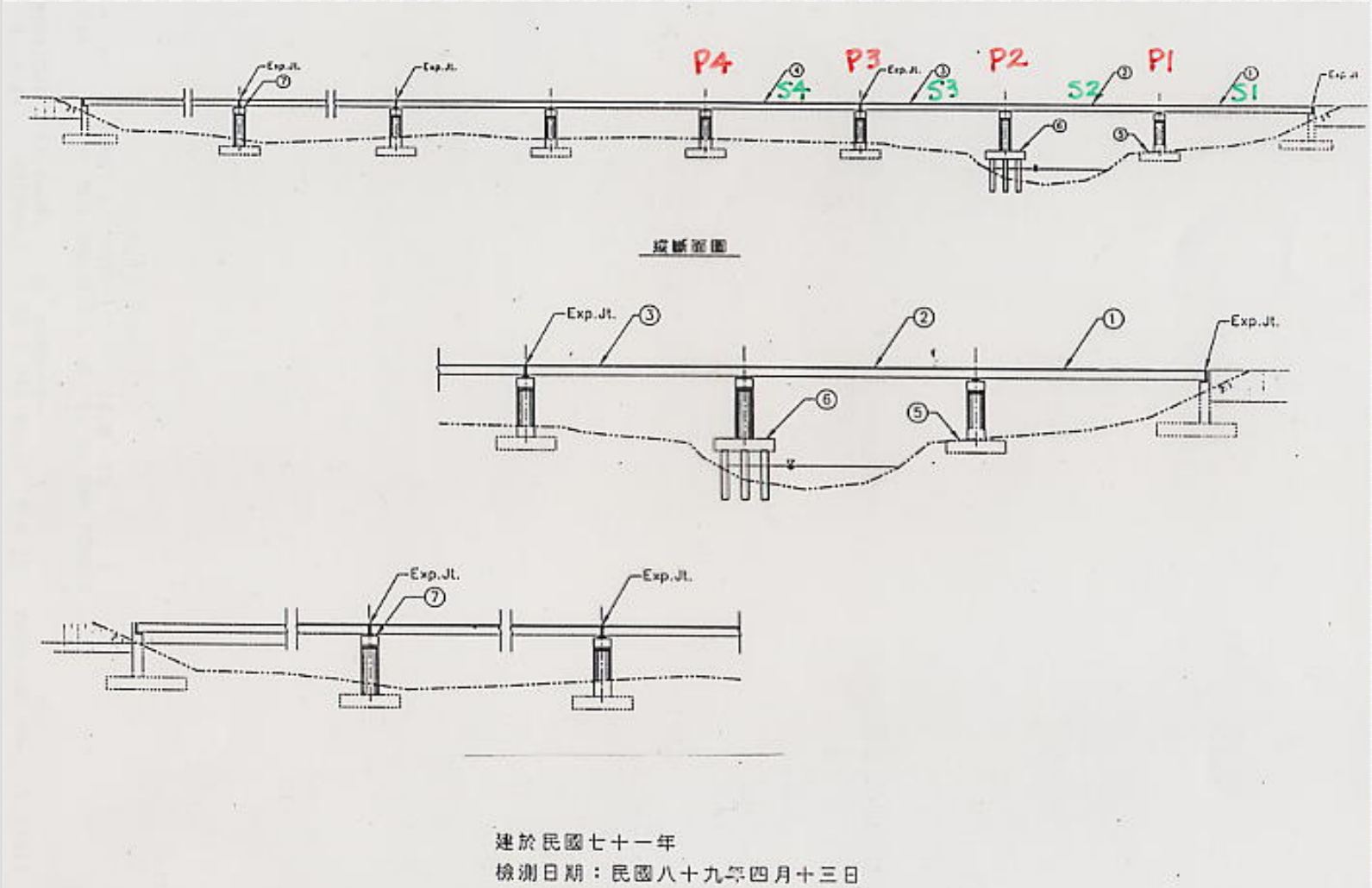
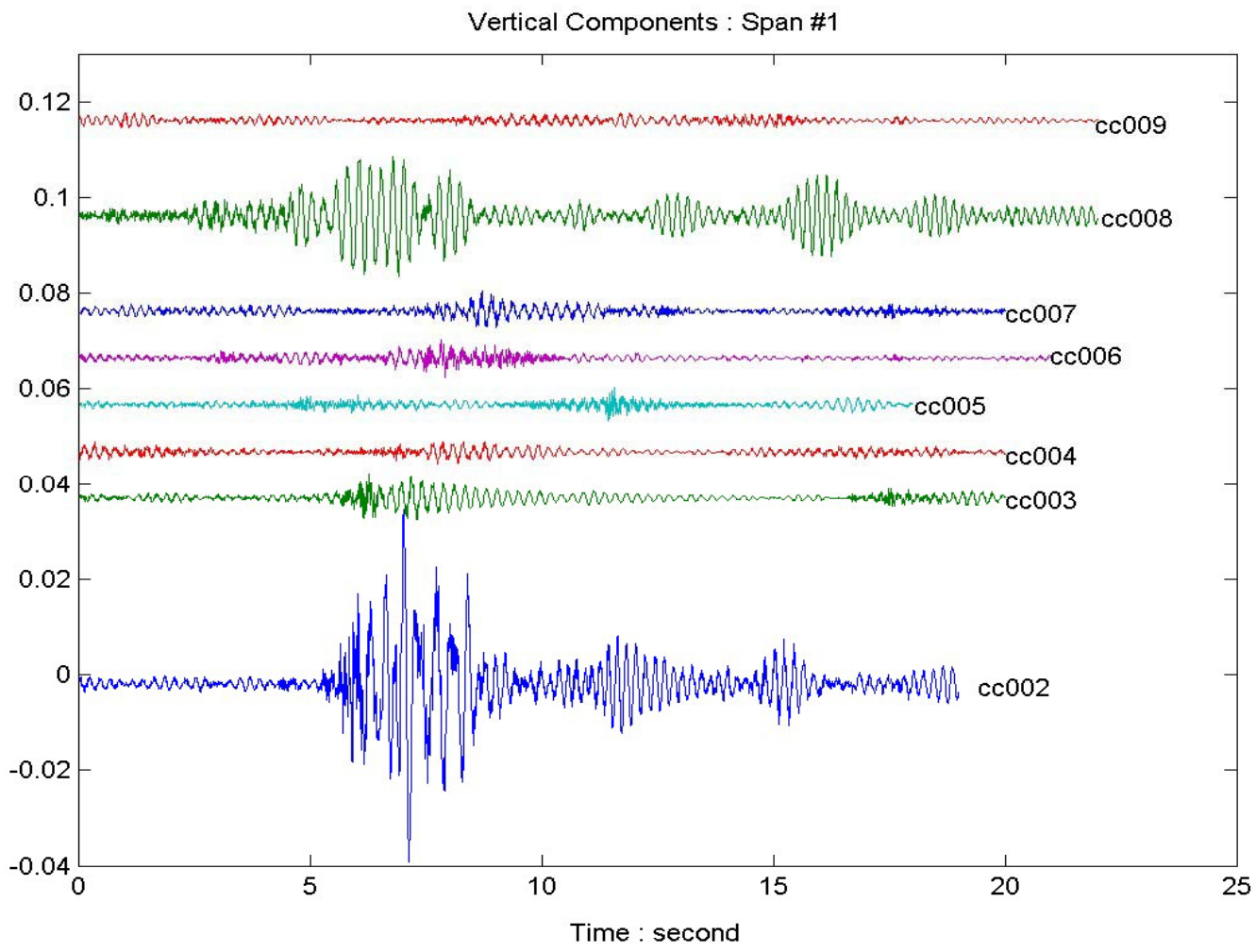
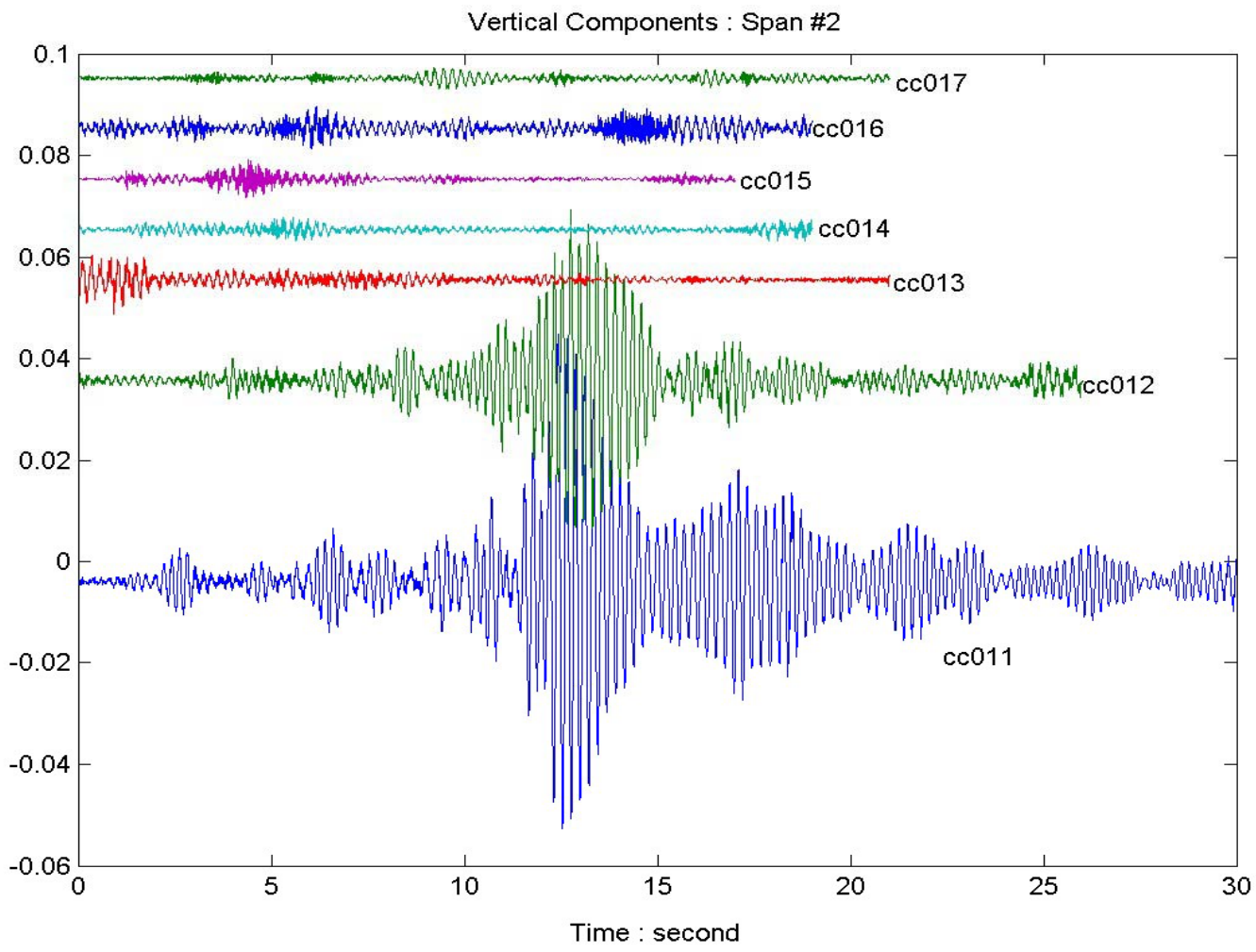
HsinNan Bridge：A sketch



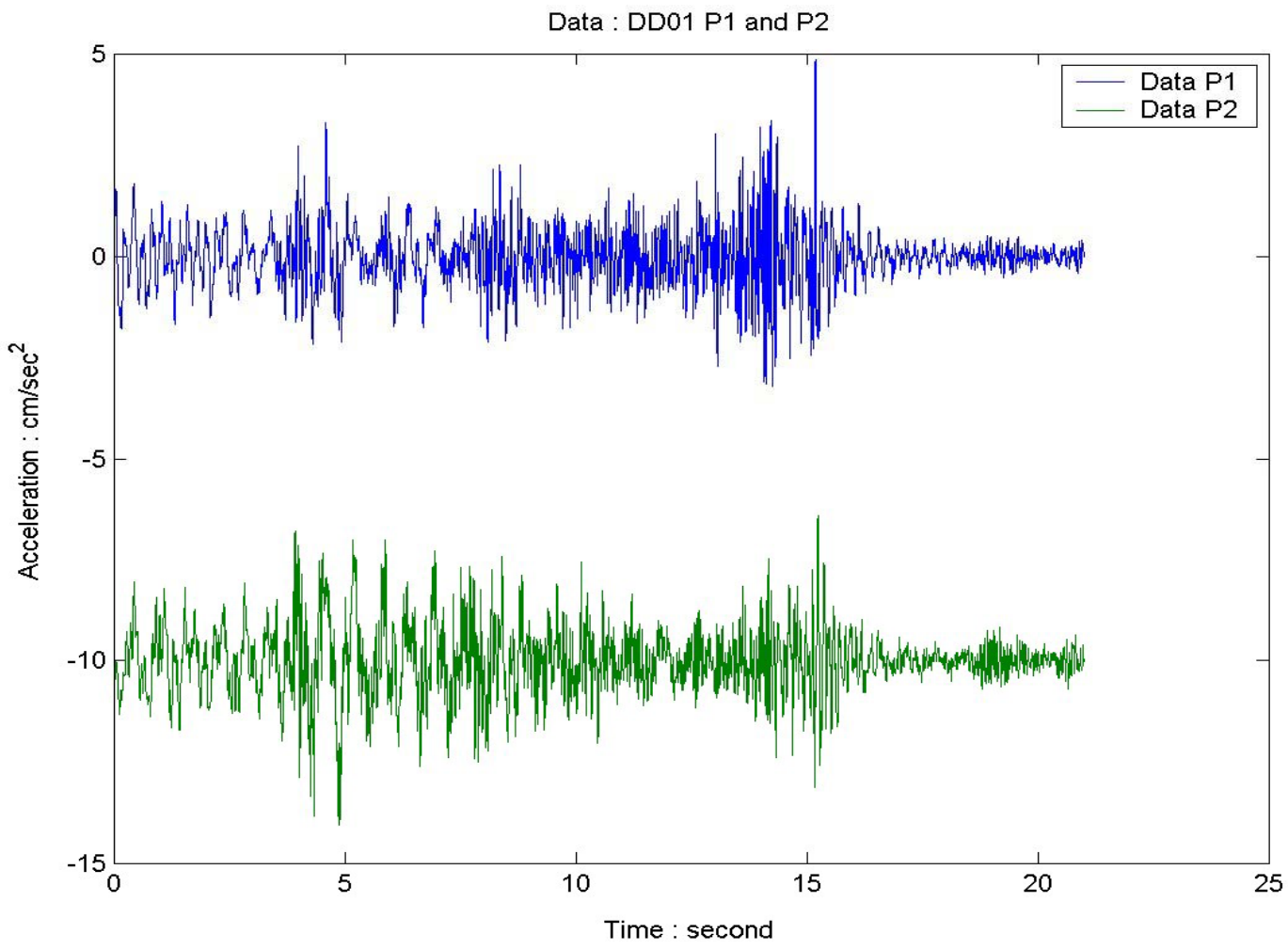
Data : Hsin Nan Bridge Span 1 Vertical



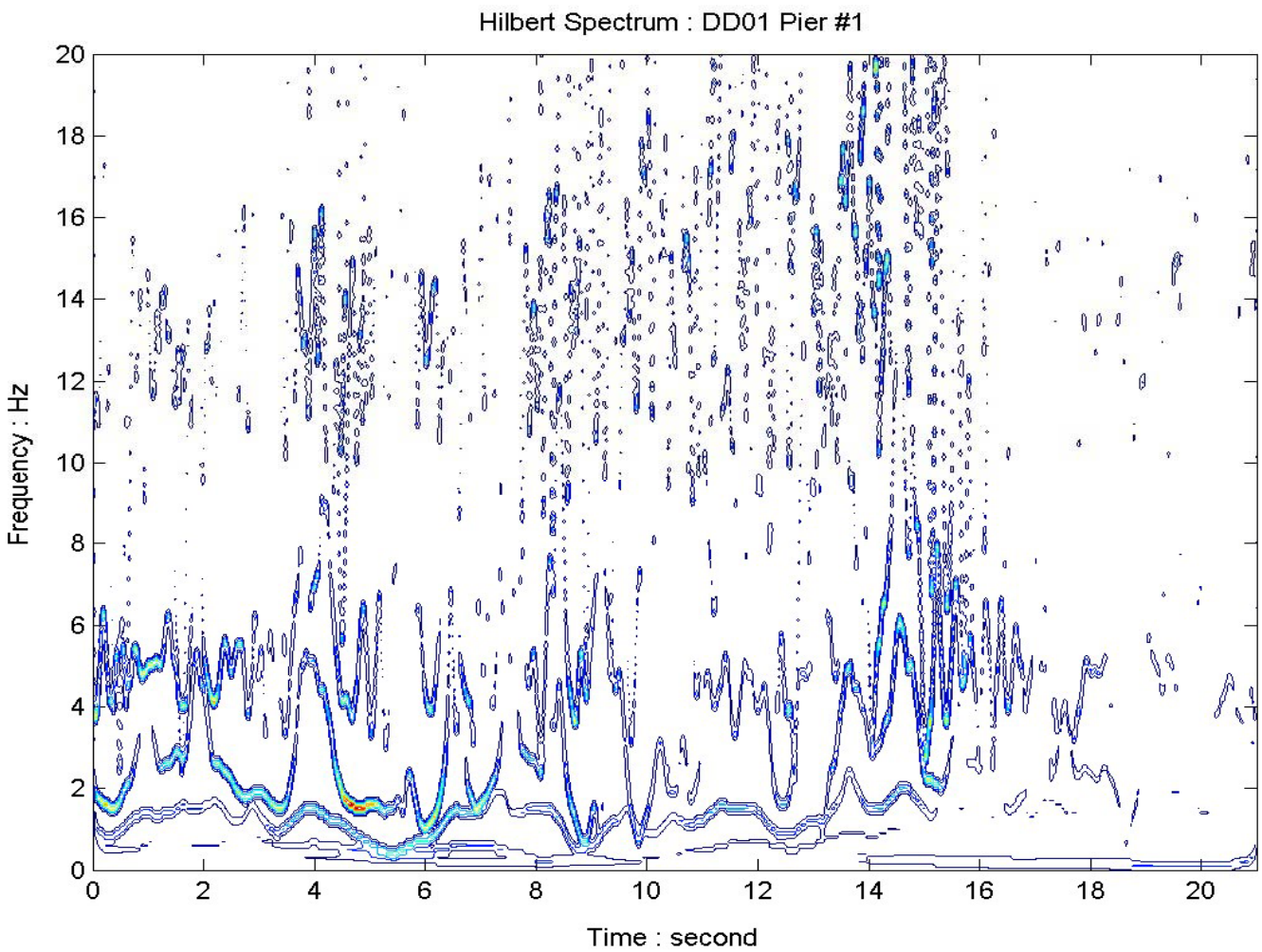
Data : Hsin Nan Bridge Span 2 Vertical



Data : Hsin Nan Bridge Piers 1 & 2

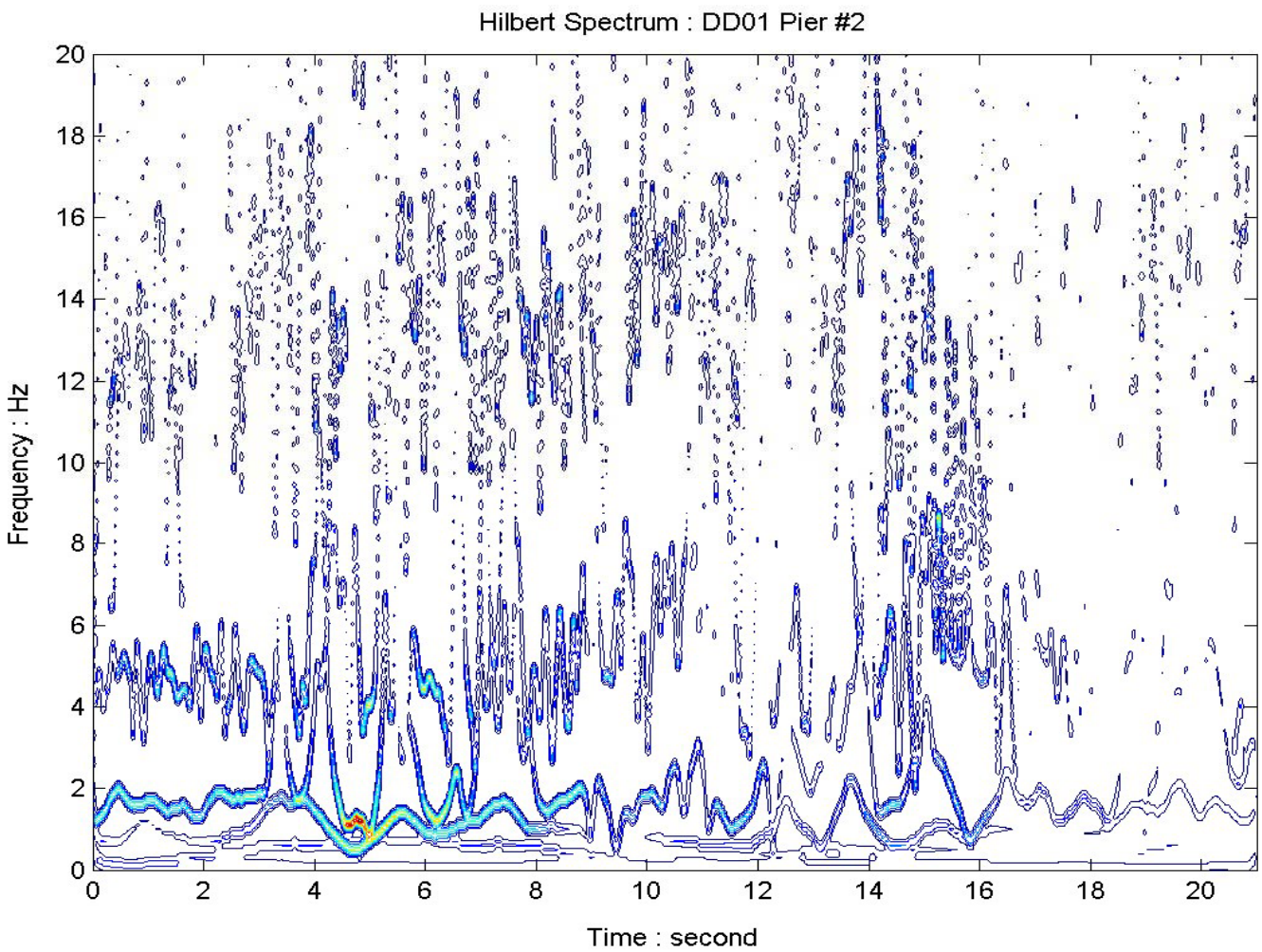


Hilbert Spectrum :
Hsin Nan Bridge Pier 1 Vertical



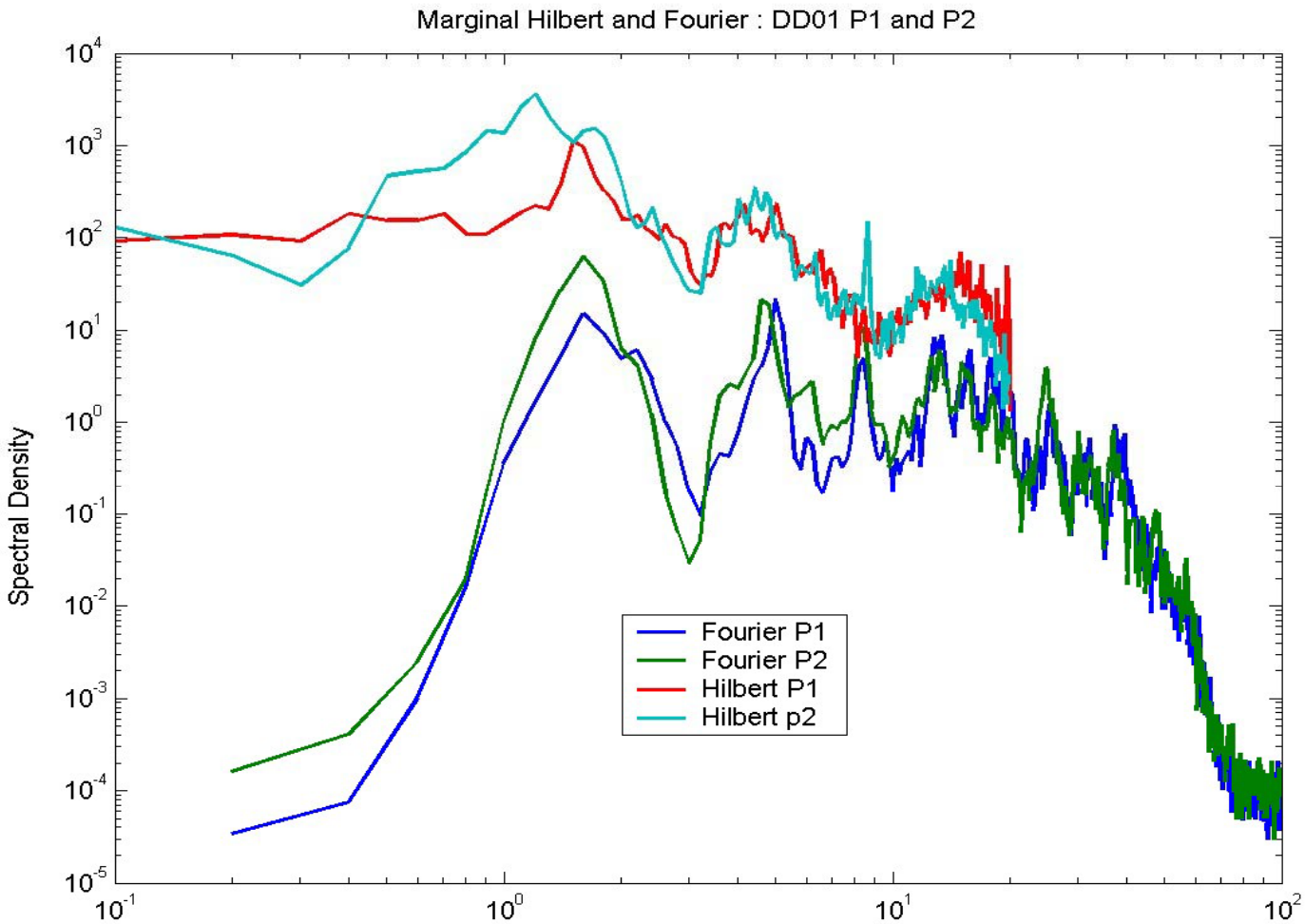
Hilbert Spectrum :

Hsin Nan Bridge Pier 2 Vertical



Marginal Hilbert & Fourier Spectra :

Hsin Nan Bridge Piers 1 & 2 Vertical



Example 7.
Airfoil Flutter Study

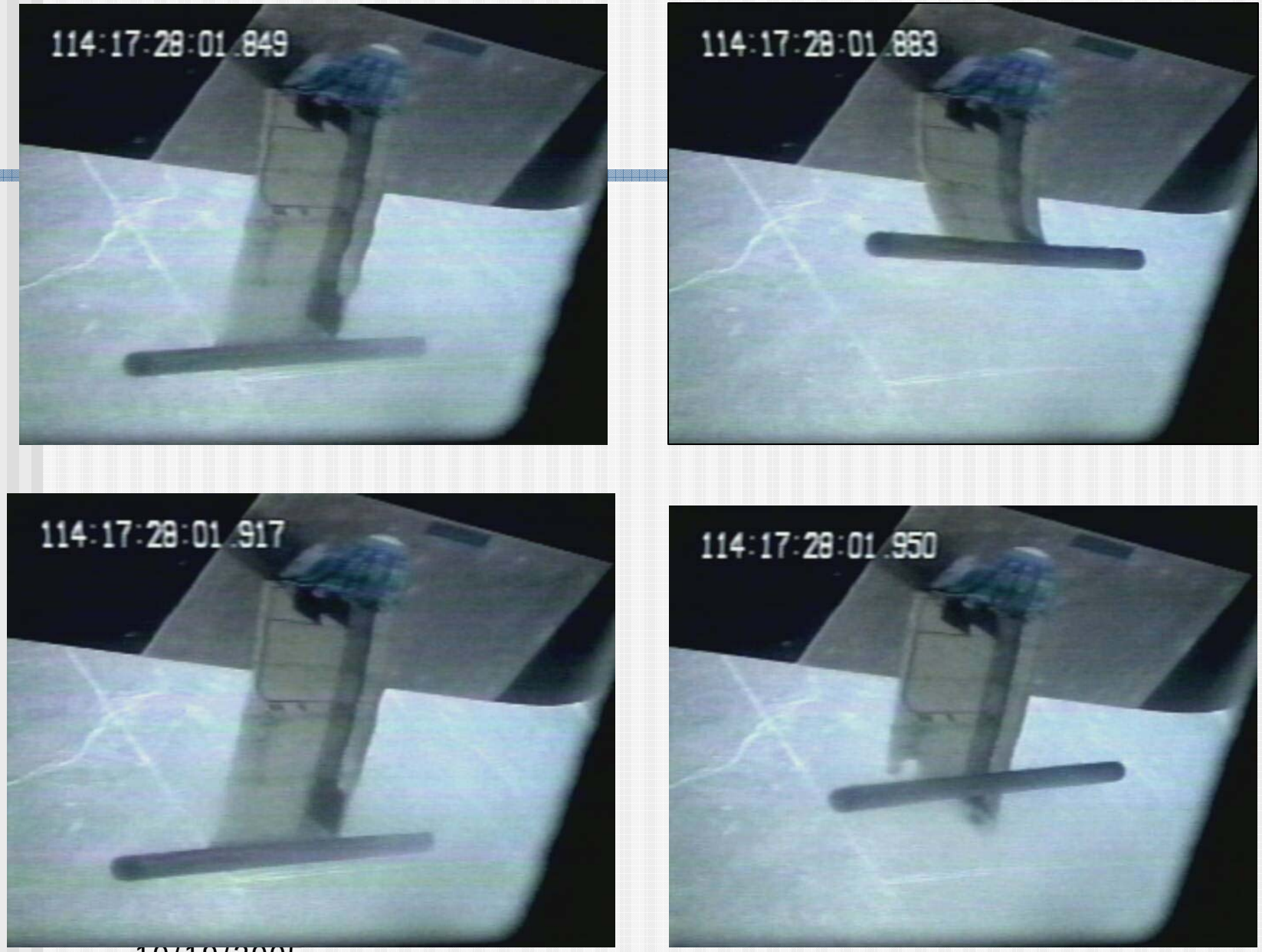
The aeroelasticity flight program to push the airfoil to the breaking point. HHT clearly identify the yield of the airfoil just before the final disintegration of the airfoil.

Fourier totally missed the critical change.

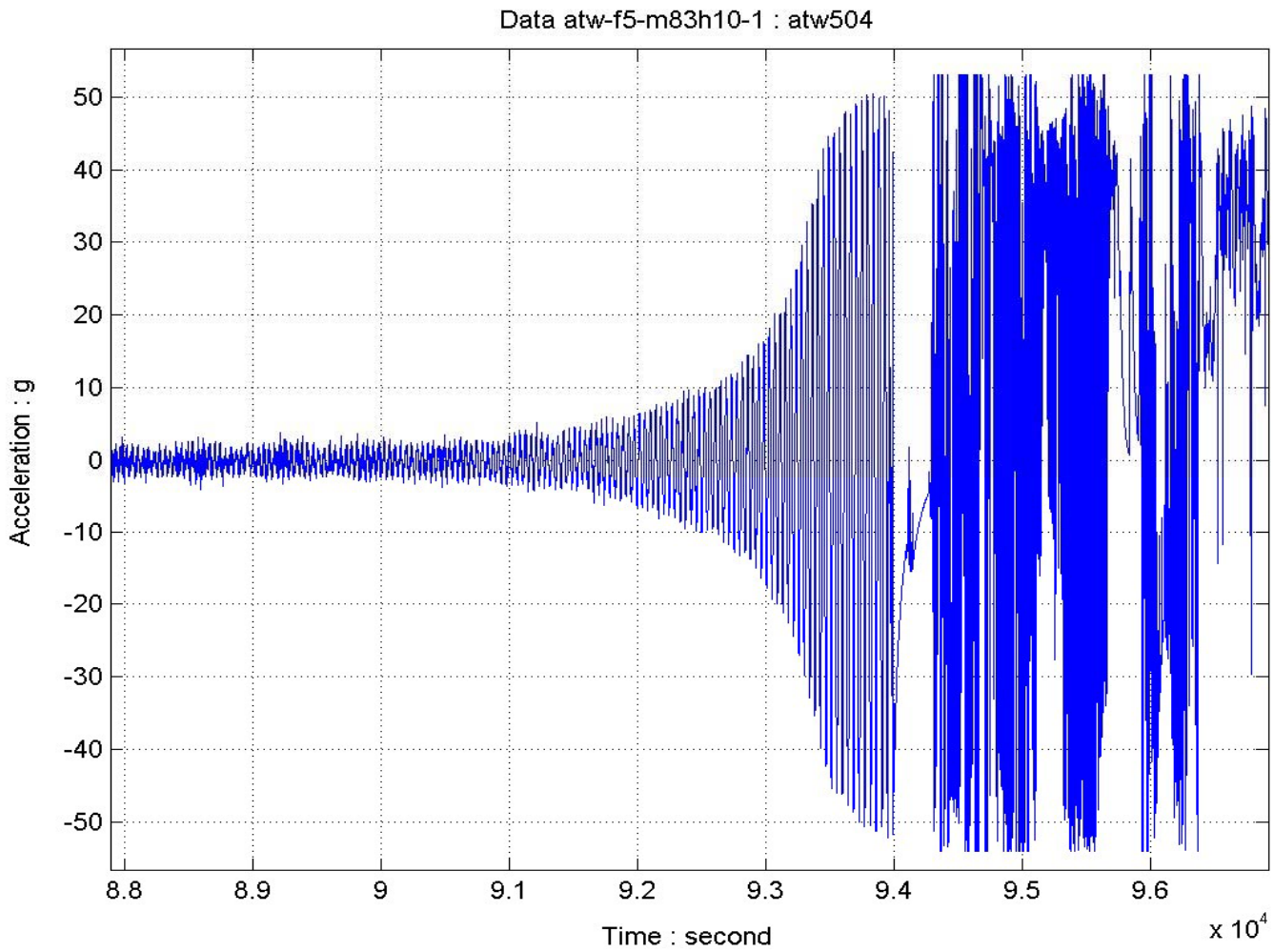
Location of the Test Wing

Details of the test wing

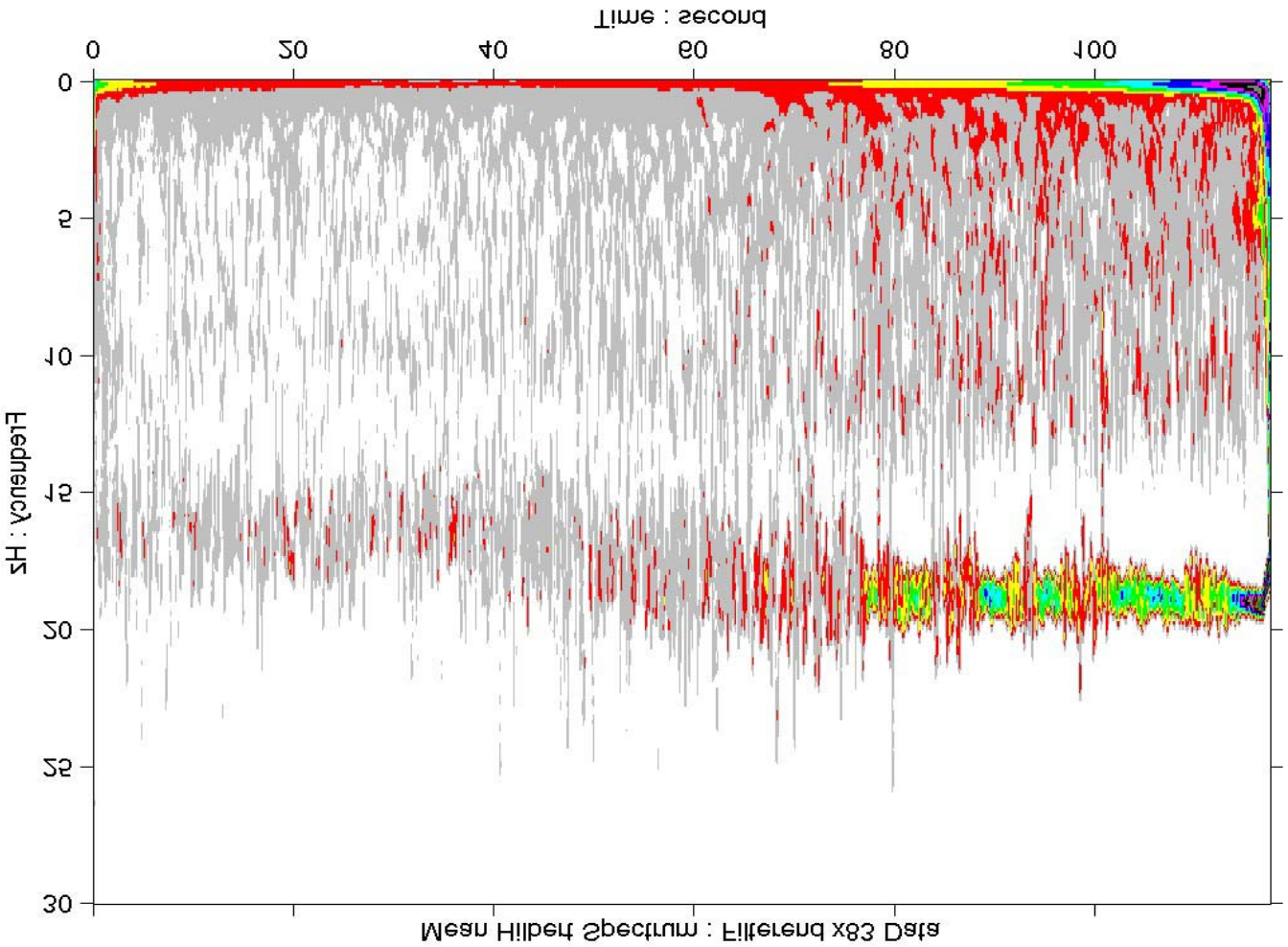
Airfoil Flutter



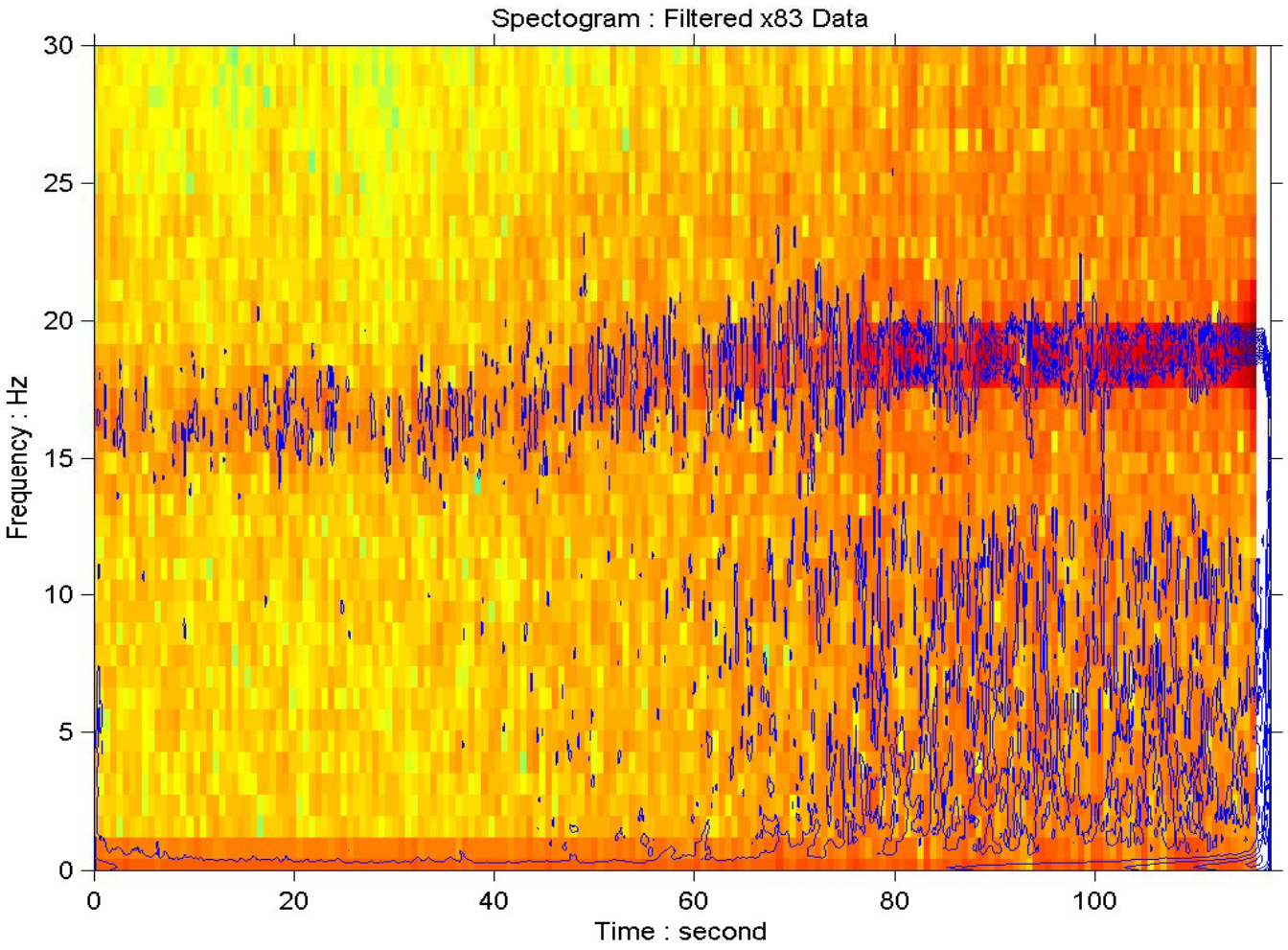
Full Data : atw_f5_m83h10_1 Details



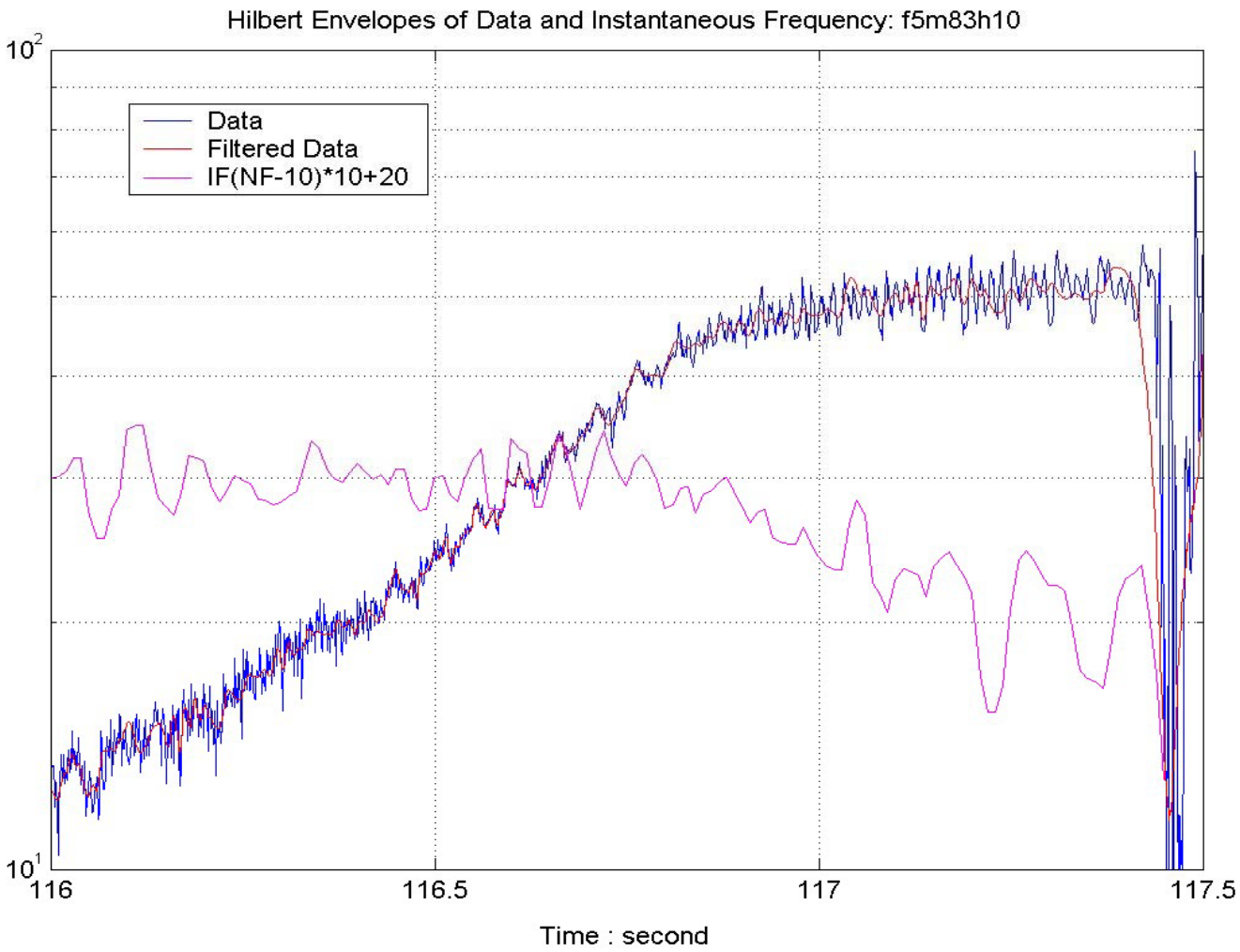
Mean Hilbert Spectrum : y(i)



Mean Hilbert and Spectrogram : y83



Instantaneous frequency and data Envelope



Example 8. Molecular Dynamics and New Drug Discovery

Work done at University of Southampton by Professor Jonathan Essex for new Drug searches

Proteins

- Polymers built from 20 monomer units
- Important in all living processes
- Structural hierarchy:
  - Primary – sequence
  - Secondary – helix or sheet
  - Tertiary – collection of secondary structure

Experimental Methods

- Timescales for these motions are at least of the order of microseconds
- Experimental methods unable to probe details of conformational changes
  - NMR, X-ray crystallography, CD
- Use molecular dynamics simulations to probe protein dynamics

Molecular Dynamics

- Empirical force field to model interactions

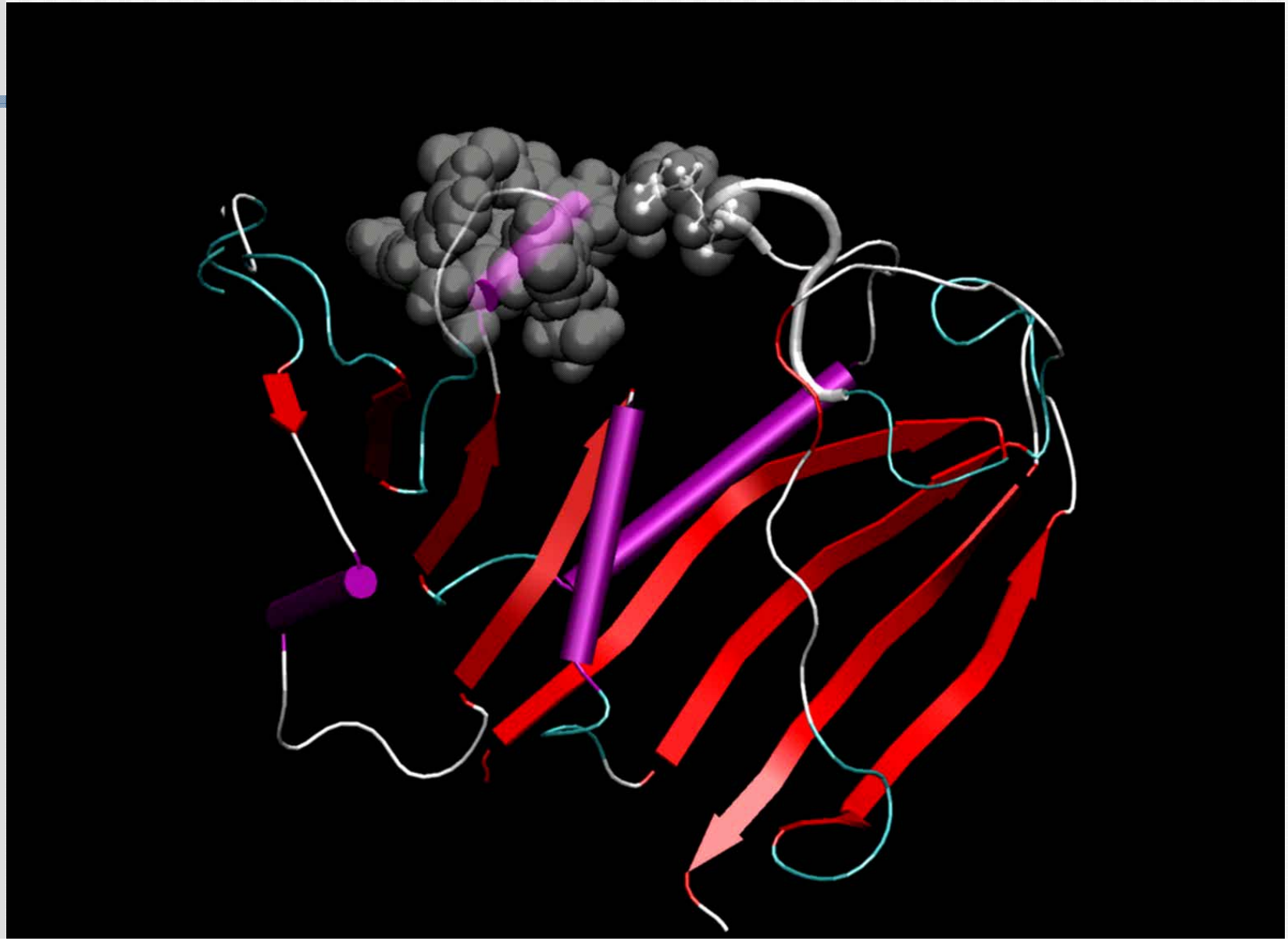
$$U = \sum_{\text{pairs}(i,j)} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} + 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$

- Solve Newton's equations of motion using finite difference approach

$$r(t+\delta t) = 2r(t) - r(t-\delta t) + \delta t^2 a(t)$$

- Time step, δt, very small (10^{-15} s)

DHFR

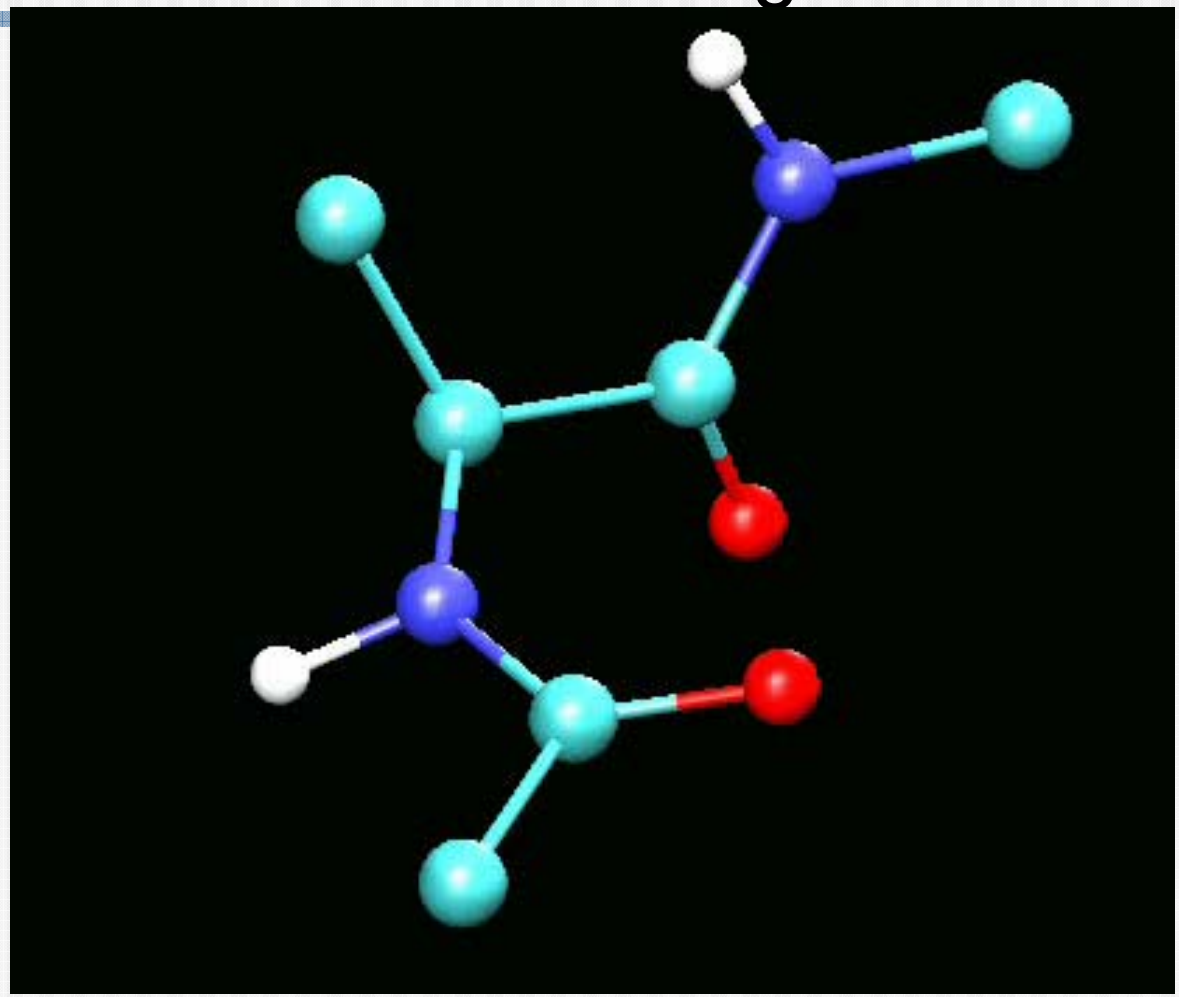


Signal Analysis of MD

- Why?
  - Simplify simulation results to aid interpretation
  - Determine frequencies of important vibrations
- Fourier transforms
- Time domain filtering using non-recursive digital filters

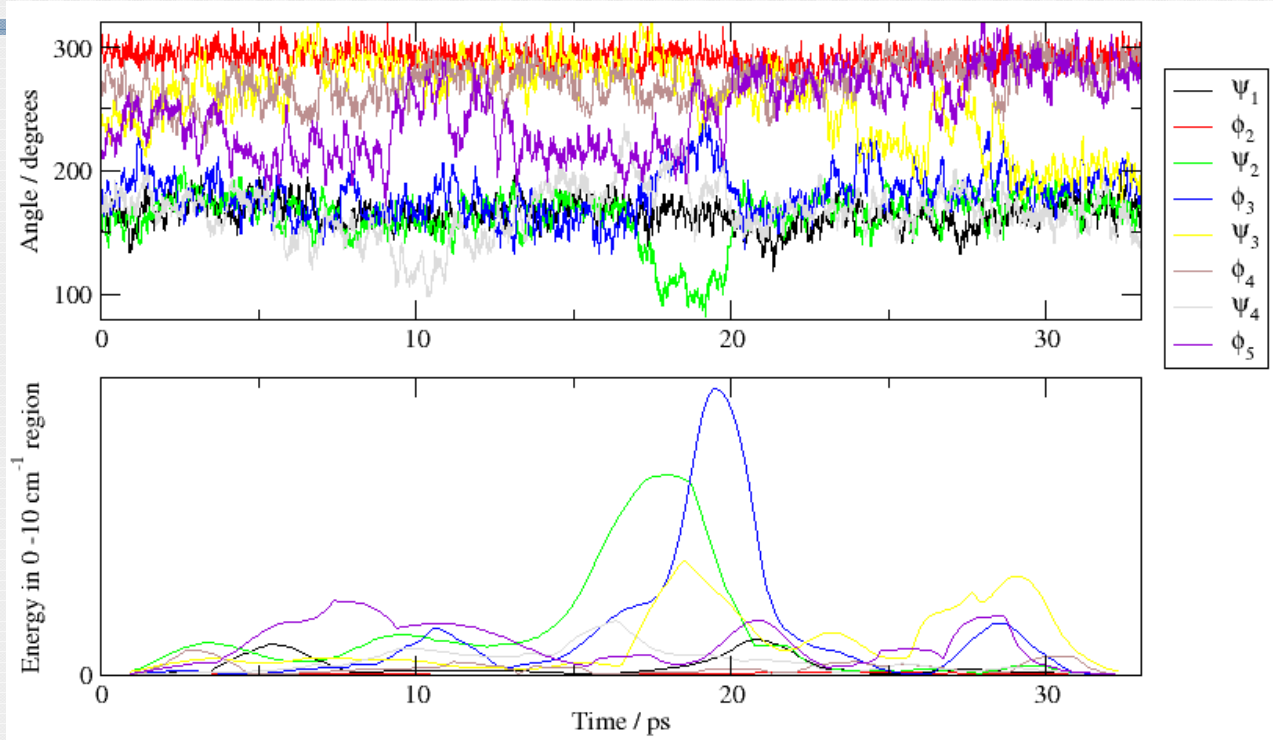
HHT & Dihedral Change

- Characterise spontaneous conformational change



HHT & Dihedral Change

- Characterise spontaneous conformational change



- Integral of HHT spectrum in 0 – 10 cm^{-1} region also shown
- Note correlations

Outstanding Mathematical Problems

1. Adaptive data analysis methodology in general
2. Nonlinear system identification methods
3. Prediction problem for nonstationary processes (end effects)
4. Optimization problem (the best IMF selection and uniqueness. Is there a unique solution?)
5. Spline problem (best spline implement of HHT, convergence and 2-D)
6. Approximation problem (Hilbert transform and quadrature)